Interferometric Polarimetry

Instrumental calibration and analysis

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VNIVERSITAT DE VALÈNCIA

OPTICON RadioNet Pilot

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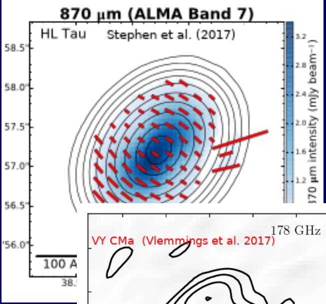
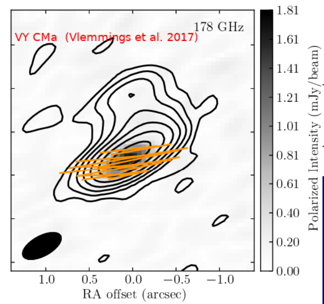
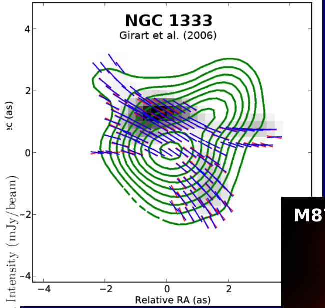
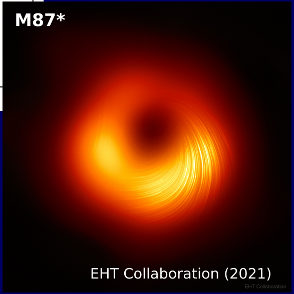
Polarized light carries a lot of information!









Polarized light in the Universe comes from very different scenarios.

Light polarization in the Universe.

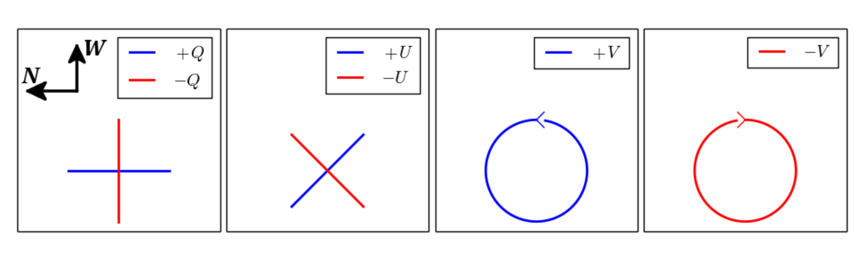
The Stokes Parameters

The Stokes parameters

- We need four quantities to fully describe the polarization state:
  - How much polarized vs. unpolarized light do we have?
  - What is the strength and orientation of the linearly polarized $\vec{E}$ field?
  - How much circular polarization do we have?

The Stokes parameters

- We need four quantities to fully describe the polarization state:
  - How much polarized vs. unpolarized light do we have?
  - What is the strength and orientation of the linearly polarized $\vec{E}$ field?
  - How much circular polarization do we have?
- The Stokes parameters: I, Q, U, and V



- Linear polarization: $I_p = \frac{\sqrt{Q^2 + U^2}}{I}$, $\theta = \frac{1}{2}\arctan\left(\frac{U}{Q}\right)$
- Unpolarized intensity: $I_u = \sqrt{I^2 - Q^2 - U^2 - V^2}$

The Poincaré Sphere

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A full rotation in azimuth corresponds to an EVPA change of 180°.

2χ is the azimuth angle;
2ϕ is the latitude:

$$\frac{Q}{I} = \cos(2\chi)\cos(2\phi)$$

$$\frac{U}{I} = \sin(2\chi)\cos(2\phi)$$

$$\frac{V}{I} = \sin(2\phi)$$

BEWARE in Astronomy!!:
Orientation convention for χ

The Poincaré Sphere

The latitude is related to the fractional circular polarization.

2χ is the azimuth angle;
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$$\frac{Q}{I} = \cos(2\chi)\cos(2\phi)$$

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BEWARE in Astronomy!!:
Sign convention for V

Polarizers in Radio Astronomy

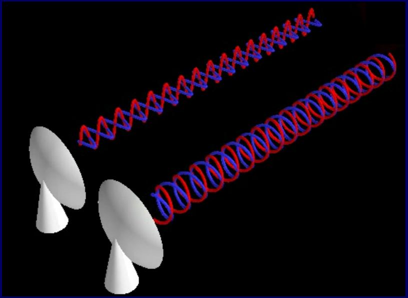
Detecting source polarization

- The Stokes parameters describe the polarization state of light. But how do we measure them?

Detecting source polarization

- The Stokes parameters describe the polarization state of light. But how do we measure them?

- Polarizing receivers (polarizers). The signal is split coherently into two orthogonal polarization states.
  - Linear polarizers (horizontal / vertical linear polarization).
  - Circular polarizers (left / right circular polarization).



Decomposing linear pol. with linear polarizers (no phase offset)

Linear polarizers

Decomposing circular pol. (left) with linear polarizers (90° offset)

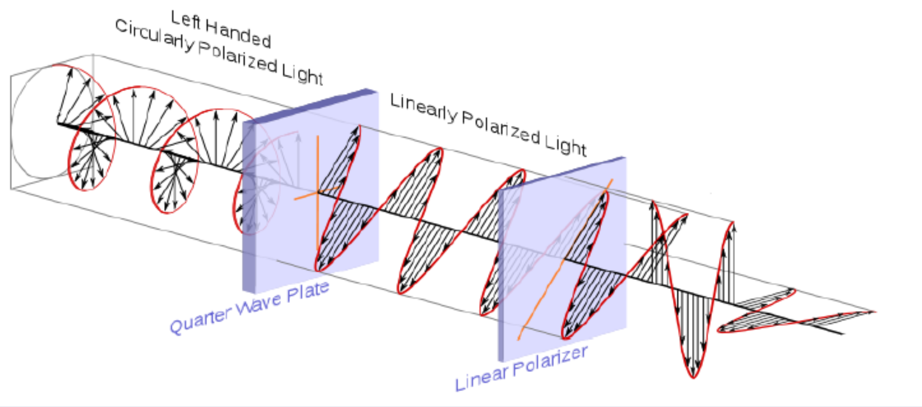
Linear polarizers

Decomposing circular pol. (right) with linear polarizers (270° offset)

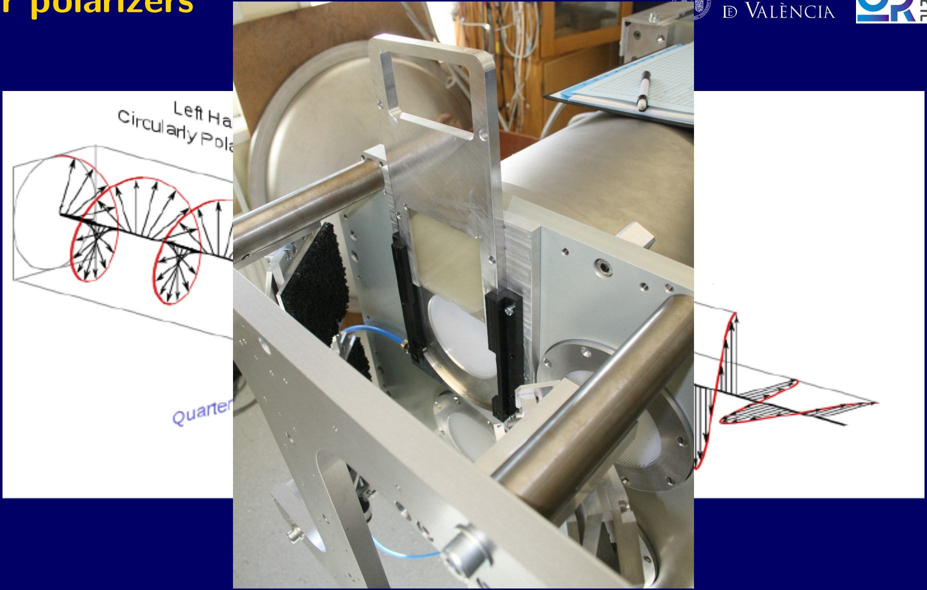
Linear polarizers

Decomposing elliptical pol. (right) with linear polarizers (any phase offset)

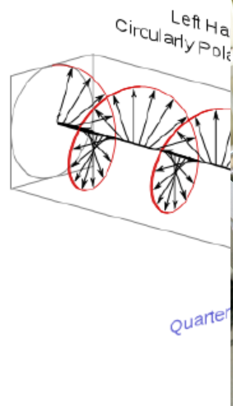
Circular polarizers

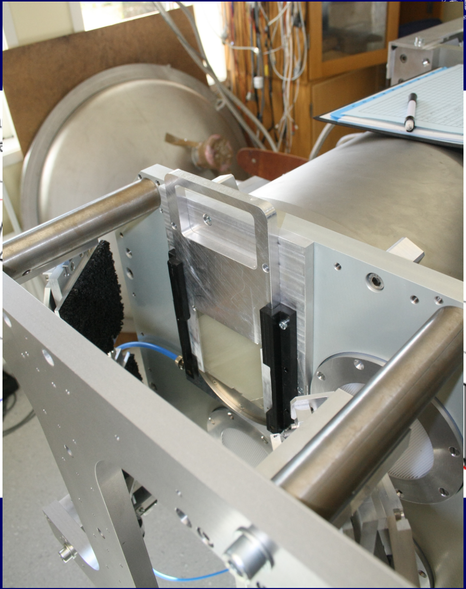


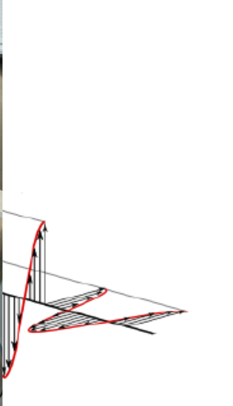
Circular polarizers



Circular polarizers







Circular polarizers

Decomposing linear pol. with circular polarizers (phase offset gives EVPA)

Circular polarizers

Decomposing elliptical pol. with circular polarizers (R/L ampl. diff.)

The Measurement Equation

Polarization and interferometry

- We measure the signal cross-correlations between radio telescopes, *a* and *b*.

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  - R^a, L^a, R^b, L^b

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- Each radio telescope registers two polarizations: R and L. Hence what we measure is:
  - R^a, L^a, R^b, L^b
- We compute all combinations of polarization cross-correlations (a.k.a. *visibilities*):
  - The so-called "parallel hands": $V_{RR}^{ab} =< R^a \times (R^b)^* >$ and $V_{LL}^{ab} =< L^a \times (L^b)^* >$.
  - The so-called "cross hands": $V_{RL}^{ab} =< R^a \times (L^b)^* >$ and $V_{LR}^{ab} =< L^a \times (R^b)^* >$

Polarization and interferometry

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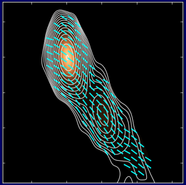
  - The so-called "cross hands": $V_{RL}^{ab} = < R^a \times (L^b)^* >$ and $V_{LR}^{ab} = < L^a \times (R^b)^* >$
- These cross-correlations can be related to the Stokes parameters of the observed source.

The RIME (e.g., Smirnov 2011)



This is what we measure:

$R^a, \quad L^a, \quad R^b, \quad L^b$

This is what we want:

$\mathcal{I}(\alpha,\delta), \quad \mathcal{Q}(\alpha,\delta), \quad \mathcal{U}(\alpha,\delta), \quad \mathcal{V}(\alpha,\delta)$

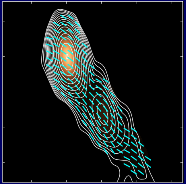
The RIME (e.g., Smirnov 2011)

This is what we measure:

$R^a, \quad L^a, \quad R^b, \quad L^b$

Visibility Matrix:

$$V_{\odot}^{ab} = \begin{bmatrix} V_{RR}^{ab} & V_{RL}^{ab} \\ V_{LR}^{ab} & V_{LL}^{ab} \end{bmatrix}$$



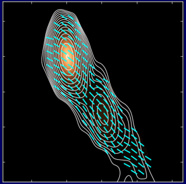
This is what we want:

$\mathcal{I}(\alpha,\delta), \quad \mathcal{Q}(\alpha,\delta), \quad \mathcal{U}(\alpha,\delta), \quad \mathcal{V}(\alpha,\delta)$

Brightness Matrix:

$$S_{\odot} = \begin{bmatrix} \mathcal{I}+\mathcal{V} & \mathcal{Q}+j\mathcal{U} \\ \mathcal{Q}-j\mathcal{U} & \mathcal{I}-\mathcal{V} \end{bmatrix}$$

The RIME (e.g., Smirnov 2011)



This is what we measure:

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Visibility Matrix:

$$V^{ab}_{\odot} = \begin{bmatrix} V^{ab}_{RR} & V^{ab}_{RL} \\ V^{ab}_{LR} & V^{ab}_{LL} \end{bmatrix}$$

Brightness Matrix:

$$S_{\odot} = \begin{bmatrix} \mathcal{I}+\mathcal{V} & \mathcal{Q}+j\mathcal{U} \\ \mathcal{Q}-j\mathcal{U} & \mathcal{I}-\mathcal{V} \end{bmatrix}$$

Radio Interferometer Measurement Equation (VLBI case):

$$V^{ab}_{\odot} = J_a \left(\int S_{\odot}(\alpha,\delta) \exp\left[2\pi j \frac{u\alpha + v\delta}{\lambda}\right] d\alpha d\delta \right) J_b^H$$

where J_a and J_b are *gain matrices*.

The MEq. A Full-Stokes Formalism

For a source with a generic structure, the visibility matrix for antennas a and b (with no direction-dependent calibration) is:

$$V^{ab} = J_a \left[\int_{\alpha,\delta} S \, e^{-\frac{2\pi j}{\lambda}(u\,\alpha + v\,\delta)} \, \frac{d\alpha\, d\delta}{z} \right] (J_b)^H,$$

Let us remember the classical equation (where V^{ab} was a *complex scalar*, not a *matrix*):

$$V^{ab} = G_a \, G_b^* \int_{\alpha,\delta} I(\alpha,\delta) \, e^{-\frac{2\pi j}{\lambda}(u\,\alpha + v\,\delta)} \, \frac{d\alpha\, d\delta}{z}$$

Jones calibration matrices. Examples

- Gain, $G = \begin{pmatrix} A_r(t)\, e^{j\phi_r(t)} & 0 \\ 0 & A_l(t)\, e^{j\phi_l(t)} \end{pmatrix}$

- Delay, $K = \begin{pmatrix} e^{j\tau_r(\nu-\nu_0)} & 0 \\ 0 & e^{j\tau_l(\nu-\nu_0)} \end{pmatrix}$

- Bandpass, $B = \begin{pmatrix} A_r(\nu)\, e^{j\phi_r(\nu)} & 0 \\ 0 & A_l(\nu)\, e^{j\phi_l(\nu)} \end{pmatrix}$

- Polarization Leakage (a.k.a. “Dterms”), $D = \begin{pmatrix} 1 & D_r(\nu) \\ D_r(\nu) & 1 \end{pmatrix}$

The Jones matrices are multiplicative, e.g.: $J = G \times B \times K$, but care must be taken, since matrices generally do not commute.

BONUS: The Closure Traces

- Broderick & Pesce (2021) found a quantity related to the *visibility matrices* that is independent of any antenna-dependent calibration effect (*i.e.*, *invariant under the effects of any (direction-independent) Jones matrix!*):

$$\mathrm{Tr}_{abcd} = \frac{1}{2}\mathrm{Tr}\left(V_{ab}V_{cb}^{-1}V_{cd}V_{ad}^{-1}\right)$$

- These quantities have some *degeneracies* (e.g., they are invariant to *rotations in the Poincaré Sphere*).

Finding Dterms!

Polarization calibration

Polarization calibration in a nutshell

- The axes of the antenna mounts are "tied" to the Earth (green). So are their polarizers.
- The source orientation is tied to the sky (yellow).
- Since the signal in a polarizer depends on its orientation w.r.t. the source, the Earth rotation allows us to decouple instrumental effects from the source polarization.

Polarization calibration

- Parallactic angle.
- Polarization leakage.
- Cross-Delay/phase.
- Amplitude ratio.

Pol. calibration I. Parallactic angle

$$P_{xy} = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \qquad P_{rl} = \begin{pmatrix} e^{j\psi} & 0 \\ 0 & e^{-j\psi} \end{pmatrix}$$

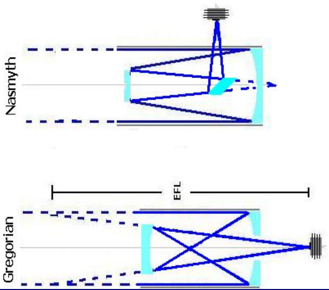
- Is the rotation of the local horizontal axis w.r.t. the sky.
- Is deterministic. It's good to apply it before the phase (and delay/rate) calibration.
- It does not commute with the gains for linear polarizers.
- In VLBI, it also mixes V_{xx} and V_{yy} with V_{xy} and V_{yx}.

Feed angle vs. Parallactic angle

- The radiation from the Sky is rotated w.r.t. the receiver polarizers with a total angle ϕ, given by:

$$\phi = \psi + \theta$$

where ψ is the parallactic angle and θ depends on the antenna mount. If E is the antenna *elevation*:



- Alt-azimuth: $\theta = 0$.
- Nasmyth left: $\theta = +E$
- Nasmyth right: $\theta = -E$
- Equatorial: $\theta = -\phi$

Pol. calibration II. Leakage

$$D_{rl} = \begin{pmatrix} 1 & D_r(\nu) \\ D_l(\nu) & 1 \end{pmatrix}$$

- Is caused by cross-talking between the polarizer channels
- Each leaked signal is modified by an amplitude and a phase (modelled with the Dterms, D).
- Introduces spurious ellipticity and linear polarization.

LIN. + LEAK

CIRC. + LEAK

Pol. calibration III. Cross-hand delay/phase

$$K_c = \begin{pmatrix} 1 & 0 \\ 0 & e^{j(\tau_c(\nu-\nu_0)+\phi_c)} \end{pmatrix}$$

- Is caused by a delay between the polarizer channels at the reference antenna.
- In linear polarizers, introduces ellipticity and spurious V.
- In circular polarizers, just rotates the PA of the linear polarization.

	OFFSET: 0°	OFFSET: 45°
LINEAR:		
CIRCULAR:		

Pol. calibration IV. Amplitude ratio

$$G_a = \begin{pmatrix} 1 & 0 \\ 0 & A_c \end{pmatrix}$$

- Is caused by different T_{sys}, gain and/or bandpass between polarizer channels.
- In linear polarizers, introduces spurious linear polarization.
- In circular polarizers, introduces spurious Stokes V.

Calibration strategy

The right order for matrix product is: $J = (G_a K_c) \times D \times P \times (G K)$

i.e.: $V^{cal} = (G K)^{-1} \times P^{-1} \times D^{-1} \times (G_a K_c)^{-1} \times V^{obs}$

Calibration strategy

The right order for matrix product is: $J = (G_a\,K_c) \times D \times P \times (G\,K)$

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- STEP 1: Calibrate the cross-delay(phase) using a strongly polarized source.

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- STEP 1: Calibrate the cross-delay(phase) using a strongly polarized source.
- STEP 2: Calibrate the leakage using an unpolarized source.
  - If all calibrators are polarized, solve for leakage and source polarization simultaneously.
  - Need good parallactic-angle coverage.

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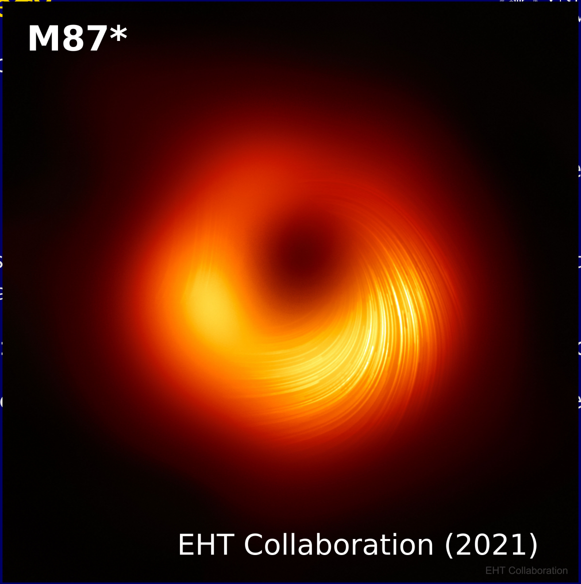
- STEP 1: Calibrate the cross-delay(phase) using a strongly polarized source.
- STEP 2: Calibrate the leakage using an unpolarized source.
  - If all calibrators are polarized, solve for leakage and source polarization simultaneously.
  - Need good parallactic-angle coverage.
- STEP 3 (optional): Refine the calibration (to minimize the "gain cross-talk").

Calibration strate

The right c ... $P \times (G\,K)$

i.e.: ... V^{obs}

- STEP 1: Calibrate ... ed source.
- STEP 2: Calibrate
  - If all calibrators ... tion simultaneously.
  - Need good para
- STEP 3 (optional): ... cross-talk").
- STEP 4: Image eac ... es: $(Q,U) \rightarrow (I_p, \theta)$.



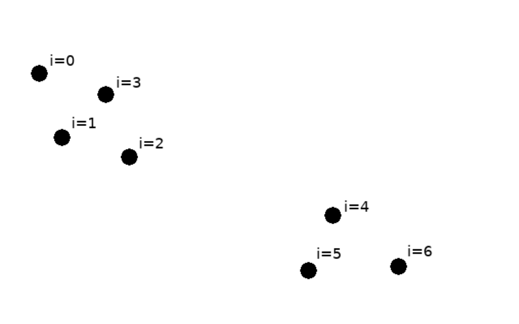
VLBI Polarimetry Software

VLBI Polarimetry Software

Nearly all the polarization calibrator sources have resolved structures in VLBI. The problem of using spatially-resolved polarization calibrators is that we need to estimate the D and (complex) $\mathcal{S}$ matrices at the same time.

- **Inverse Modelling.**
  - **LPCAL** (Leppänen et al. 1995) for AIPS. Pretty old, but well established and tested.
  - **GPCAL** (Park et al. 2020) for AIPS. Overcomes some LPCAL limitations.
  - **PolCal** (Moellenbrock) for CASA. Some limitations critical for VLBI.
  - **PolSolve** (Marti-Vidal et al. 2020) for CASA. Overcomes some LPCAL limitations.
- **Forward Modelling.**
  - **EHTim** (A. Chael et al. 2018, 2020)
- **MCMC.**
  - **DMC** (D. Pesce 2020) and **THEMIS** (Broderick et al. 2020)

Resolved Calibrator Approach I: Similarity



$$\mathcal{V}_I(u,v) = \sum_i I_i e^{2\pi j (u\alpha_i + v\delta_i)}$$

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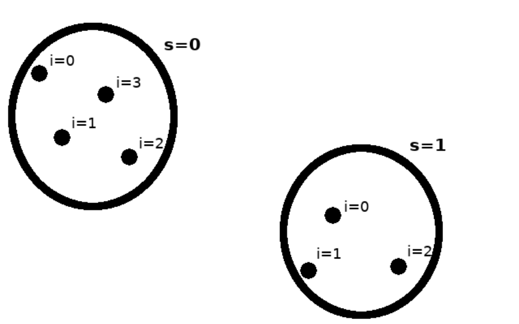
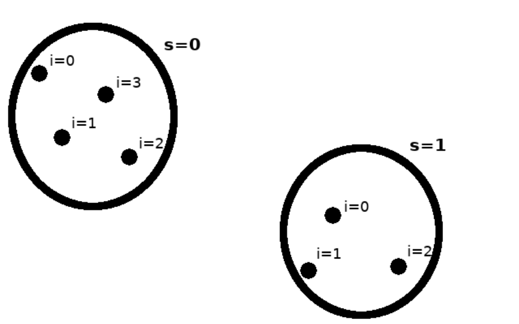


$$\mathcal{V}_I(u,v) = \sum_s \sum_i I_i^s e^{2\pi j (u\alpha_i^s + v\delta_i^s)}$$

- We divide the CLEAN components into *disjoint subsets* of constant fractional polarization.

Resolved Calibrator Approach I: Similarity



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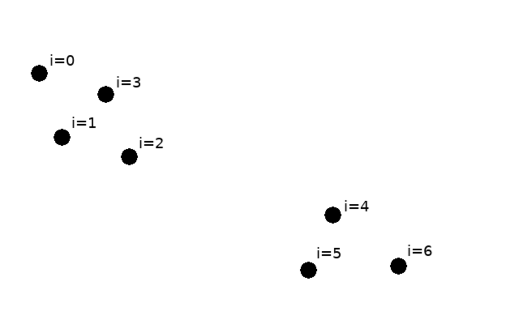
$$\mathcal{V}_Q(u,v) = \sum_s q_s \sum_i I_i^s e^{2\pi j (u\alpha_i^s + v\delta_i^s)} \qquad \mathcal{V}_U(u,v) = \sum_s u_s \sum_i I_i^s e^{2\pi j (u\alpha_i^s + v\delta_i^s)}$$

- We divide the CLEAN components into *disjoint subsets* of constant fractional polarization.

Resolved Calibrator Approach II: Selfcal



$$\mathcal{V}_I(u,v) = \sum_i I_i e^{2\pi j (u\alpha_i + v\delta_i)}$$

$$\mathcal{V}_Q(u,v) = \sum_i Q_i e^{2\pi j (u\alpha_i + v\delta_i)} \quad \mathcal{V}_U(u,v) = \sum_i U_i e^{2\pi j (u\alpha_i + v\delta_i)}$$

- We CLEAN (in full pol.) and estimate Dterms *iteratively*.

The PolSolve Algorithm

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- Fits the Dterms (and source polarization) using the full Meq.
- Computes the error function *in the Receiver frame*.

$$\chi^2 = \sum_{i,pol} W_i \left(\mathcal{V}^{Rec}_{mod,pol} - \mathcal{V}^{Rec}_{obs,pol} \right)^2_i$$

where *pol* can be *RL*, *LR*, *RR* and *LL*; and (for visibilities with baseline *a*-*b*):

$$\mathcal{V}^{Rec}_{mod,RL}(u,v) = \left(\sum_s q_s I^s_{mod} + j \sum_s u_s I^s_{mod} \right) (e^{-j\delta}) + ((D_R)_a + (D_L)^*_b)\, \mathcal{V}^{Rec}_{obs,I} + \mathcal{O}(D^2)$$

and similarly for LR.

- Includes: multi-source calibration, wide-band modelling, linear polarizers in VLBI, etc.

SUMMARY

- We have reviewed basic concepts of polarization.
  - Modes of polarization.
  - Stokes parameters.
- We have discussed about the different kinds of polarizers in radioastronomical receivers.
  - Linear polarizers (X-Y).
  - Circular polarizers (R-L).
- We have studied how to deal with polarization in interferometric observations.
  - The Measurement Equation.
  - The matrices for polarization calibration.
  - Calibration effects on X-Y vs. R-L polarizers.
  - Overview of calibration procedure.

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Now, it's time to play with some data!

Tutorial Scope

1. Installation of PolSolve (GNU/Linux & Mac).
2. Data simulation and inspection.
   - Visibilities and closure traces.
3. Simple case (unresolved calibrator): PolCal vs. PolSolve.
4. PolSolve on resolved calibrator.
5. Real Data (VGOS PolConverted visibilities).

BONUS SLIDES

Polarizers and Stokes Parameters

Circular Polarizers:

- $I = |E_l|^2 + |E_r|^2$
- $V = |E_l|^2 - |E_r|^2$
- $Q = 2\,Re(E_l^*\,E_r)$
- $U = -2\,Im(E_l^*\,E_r)$

Linear Polarizers:

- $I = |E_x|^2 + |E_y|^2$
- $Q = |E_x|^2 - |E_y|^2$
- $U = 2\,Re(E_x\,E_y^*)$
- $V = 2\,Im(E_x\,E_y^*)$

The LPCAL Algorithm

- Fits the Dterms (and source polarization) using a linear approximation.
- Computes the error function *in the Sky frame*.

$$\chi^2 = \sum_{i,pol} W_i \left(\mathcal{V}^{Sky}_{mod,pol} - \mathcal{V}^{Sky}_{obs,pol} \right)^2_i$$

where *pol* can be *RL* and *LR*, and (for visibilities with baseline *a*-*b*):

$$\mathcal{V}^{Sky}_{mod,RL}(u,v) =$$

$$\sum_s q_s \sum_k I^s_k e^{2\pi j (u\alpha^s_k + v\delta^s_k)} + j \sum_s u_s \sum_k I^s_k e^{2\pi j (u\alpha^s_k + v\delta^s_k)} + \left((D^{Sky}_R)_a + (D^{Sky}_L)^*_b \right) \mathcal{V}^{Sky}_{obs,I}$$

and similarly for LR. The fitting parameters are q_s, u_s, and the Dterms.

- Reduces the polarization calibration to a *linear least-squares* problem.

LPCAL Frame for the Dterms

The visibility matrix is

$$\mathcal{V} = \begin{pmatrix} RR^* & RL^* \\ LR^* & LL^* \end{pmatrix}$$

And the brightness matrix is

$$\mathcal{S} = \begin{pmatrix} I+V & Q+jU \\ Q-jU & I-V \end{pmatrix}$$

Receiver Frame and Sky Frame are related by a Rotation matrix: $P(\phi) = \begin{pmatrix} e^{j\phi} & 0 \\ 0 & e^{-j\phi} \end{pmatrix}$

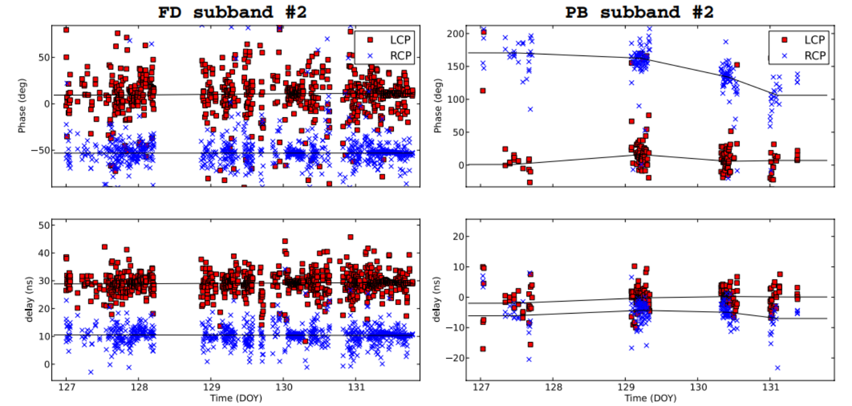
$$\mathcal{V}_{ab}^{Sky} = P(\phi^a)\, \mathcal{V}_{ab}^{Rec}\, P(-\phi^b)$$

$$\mathcal{S}_{ab}^{Sky} = P(\phi^a)\, \mathcal{S}_{ab}^{Rec}\, P(-\phi^b) = \begin{pmatrix} (I+V)e^{j\delta} & (Q+jU)e^{j\Delta} \\ (Q+jU)e^{-j\Delta} & (I+V)e^{-j\delta} \end{pmatrix}$$

where $\Delta = \phi^a + \phi^b$ and $\delta = \phi^a - \phi^b$ $\quad\rightarrow\quad$ Dterms and feed angle are coupled.

Classical VLBI Polarization Calibration

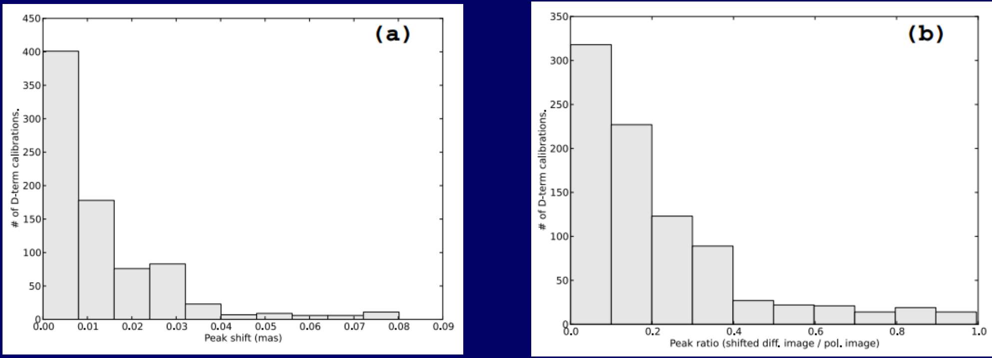
- R-L phase stability is assumed.
- Only first-order D-term effects are considered.
- Absolute EVPA is unknown *per sé*.



Cross-pol. phases can vary on day timescales (Martí-Vidal et al. 2012)

Classical VLBI Polarization Calibration

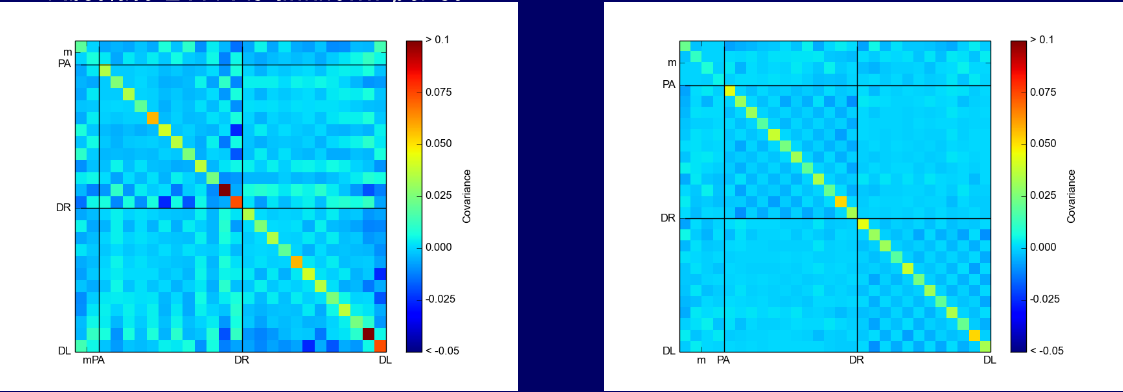
- R-L phase stability is assumed.
- Only first-order D-term effects are considered.
- Absolute EVPA is unknown *per sé*.



Heavy pol. cutoffs required to deal with residual D-term effects (Marti-Vidal et al. 2012)

Classical VLBI Polarization Calibration

- R-L phase stability is assumed.
- Only first-order D-term effects are considered.
- Absolute EVPA is unknown *per sé*.



D-term covariance matrix for one calibrator (left) and 2 calibrators (right), with the same total observing time (Marti-Vidal 2016).