Introduction to Interferometry and Calibration

George Moellenbrock, NRAO

CASA VLBI Workshop
2023 June 5-9

submillimeter Array
ed Very Large Array
een Bank Telescope
Long Baseline Array

Synopsis

- Image formation and the camera obscura
- Focus, resolution, and segmented apertures
- Young's interference
- Signal arrival geometry, mutual incoherence of sources, and the Van Cittert-Zernike Theorem
- Antenna-based calibration
- Calibration factorization and topic previews
- Summary

→ An intuition-driven description of a general image-formation process

- *Synthesis interferometry, especially VLBI, is doing an experiment, and this involves understanding, controlling, and characterizing you apparatus. With interferometry data, we are afforded uncommon access to the fine-tuning of effective focus, and with great power comes great responsibility. Understanding you instrument is (at least!) half the fun.*

References

- for rigor and details…

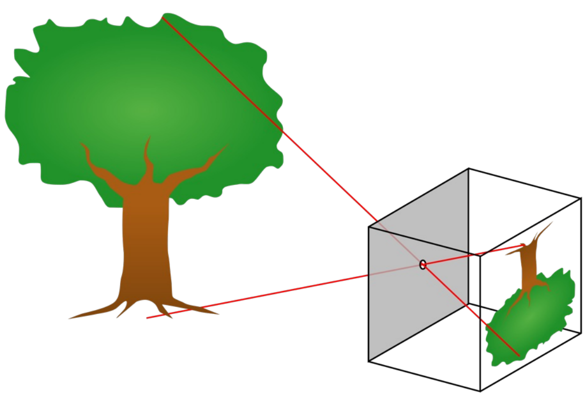
- Synthesis Imaging in Radio Astronomy II (Editors: Taylor, Carilli, & Perley)
- Interferometry and Synthesis in Radio Astronomy (3rd ed., Thompson, Moran, & Swenson)
- Tools of Radio Astronomy (6^{th} ed., Wilson, Rohlfs, & Huettemeister)
- The Fourier Transform and its Applications (Bracewell)

What is *image formation?*

- We wish to record images of (regions of) the sky, to study the physics and behavior of objects in the universe.
- …which is to say: *isolate the direction of arrival of light--EM radiation--with as much **resolution** and **sensitivity** as possible, and in a persistent form, so that we can discern and study the objects and processes generating the radiation.*
  - EM radiation is the principal "messenger" but there are others: cosmic rays, gravitational radiation…
- However, any point in space where we might attempt such a measurement is bathed in EM radiation from all directions…
  - Evidently (and intuitively), EM radiation travels from objects in ~straight lines (spreading out), even over vast distances
  - E.g., objects cast shadows, a very rudimentary ~indirect "image" possibly revealing multiple sources of radiation…
  - We must somehow *organize* the incoming light to form an image

Camera obscura (pin-hole camera)

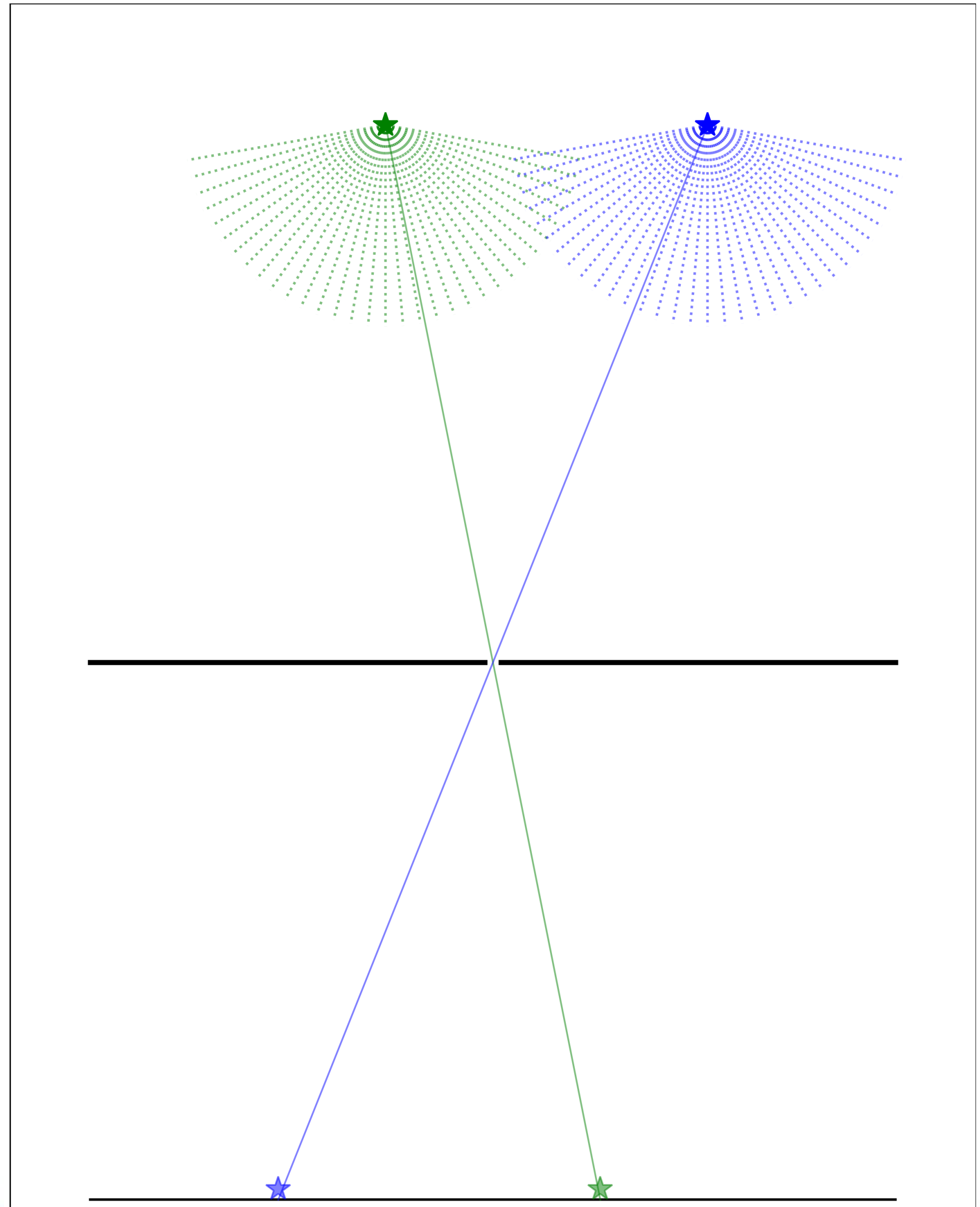
- Earliest instance of deliberate and true direct image formation
- Permitting light to enter a darkened room via a single small opening necessarily and implicitly selects direction of arrival in formation of an (inverted) image on opposite wall.
- Requires very brightly-illuminated subject, otherwise very dim
- Not practical for astronomy (except for sun!)



NRAO

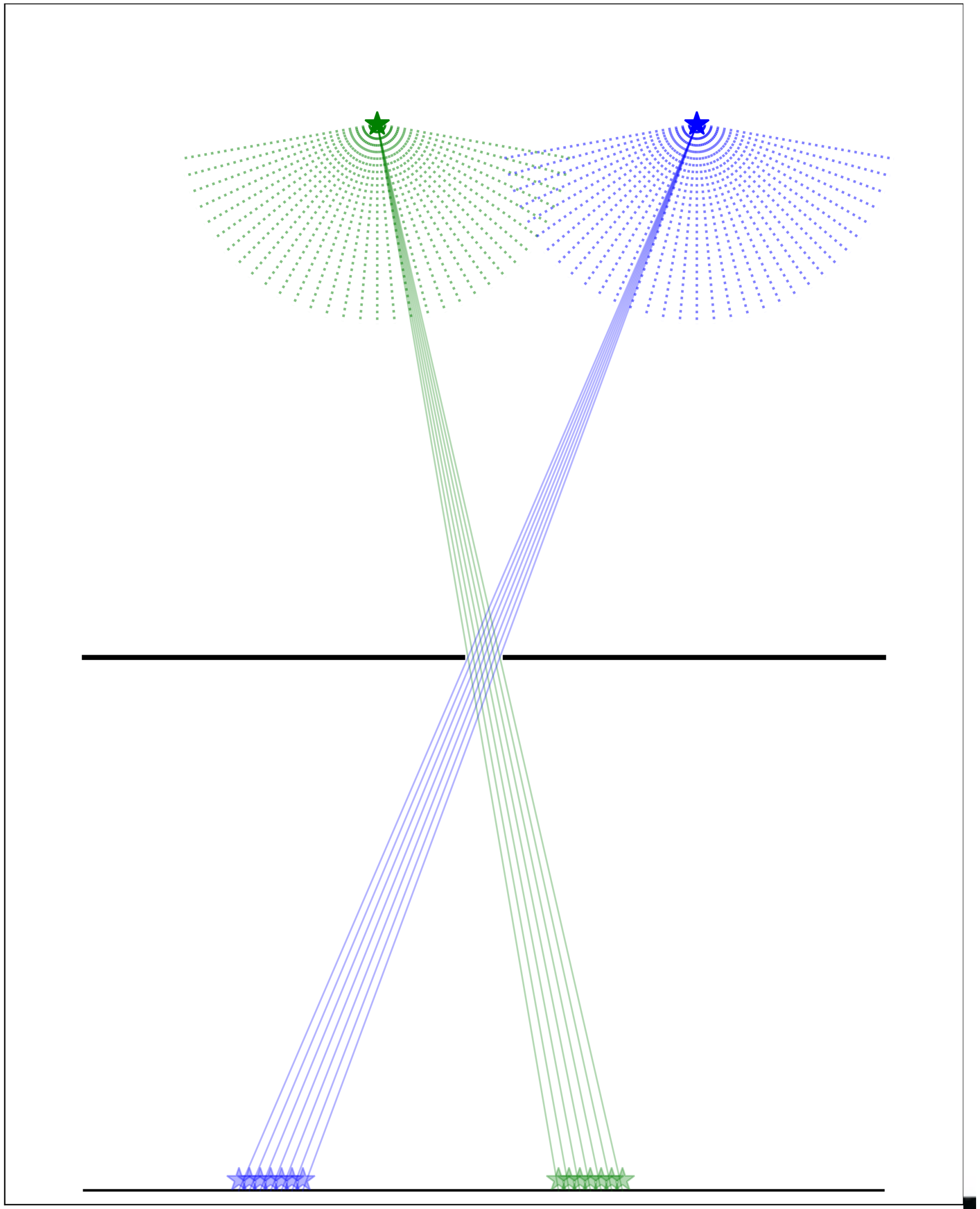
How to improve Camera Obscura?

- Tiny aperture: dim image



How to improve Camera Obscura?

- Larger aperture, to let through more light
  - But images smeared



How to improve Camera Obscura?

- Larger aperture, to let through more light
- Add a lens to *focus* it (c.f., our eyes!)
  - Can also use a curved mirror

- → Cameras (including in phones!), photography, and a world full of optical images
- → Hubble Space Telescope, JWST, etc.

- High-precision optics—ensuring good focus over field-of-view — can be expensive

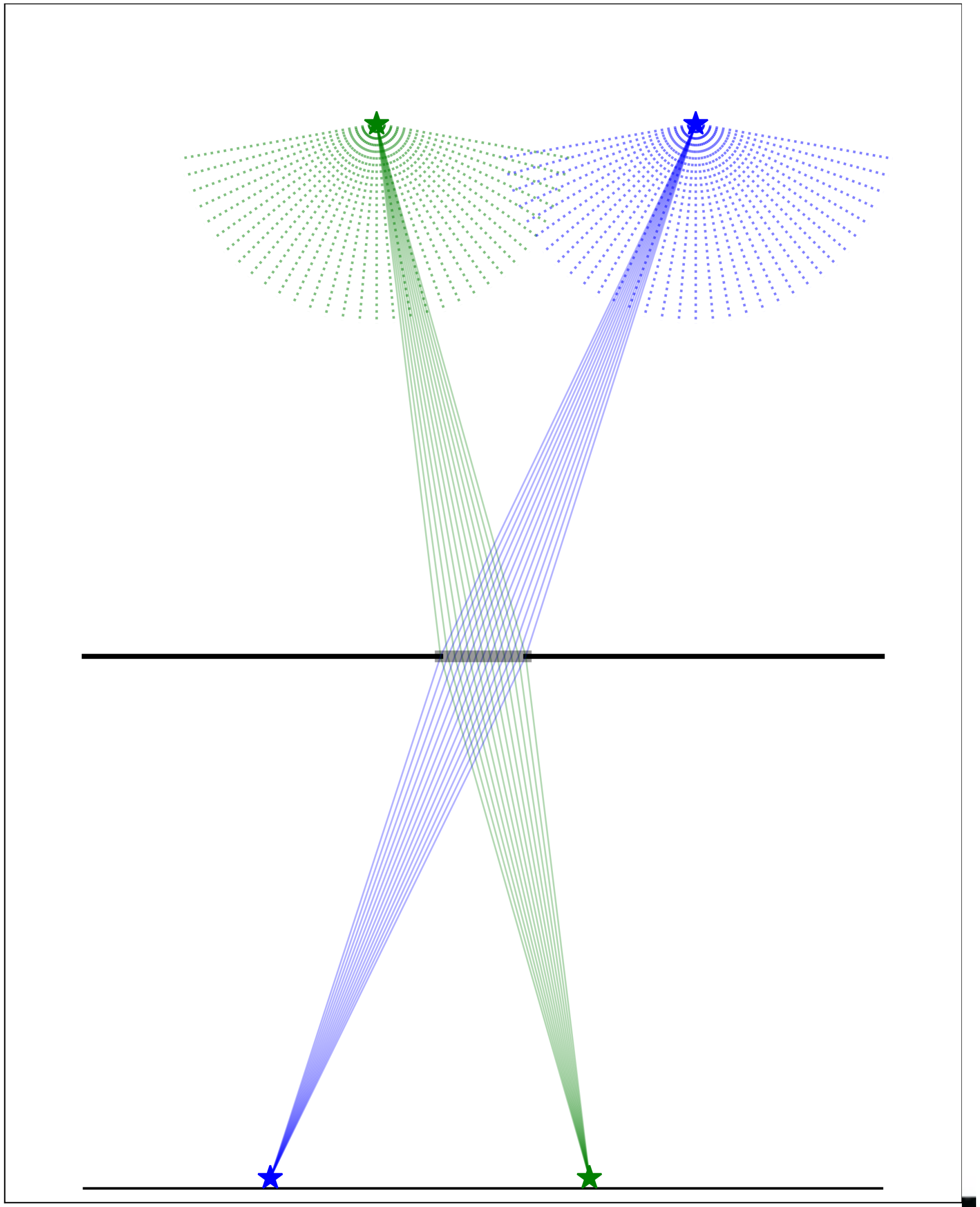
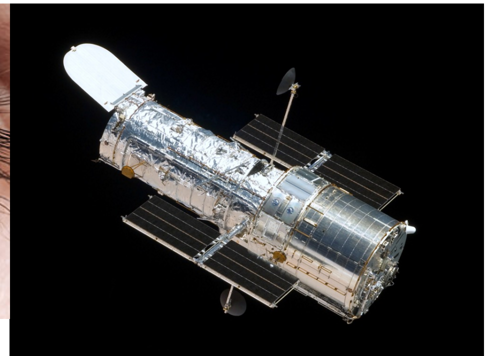
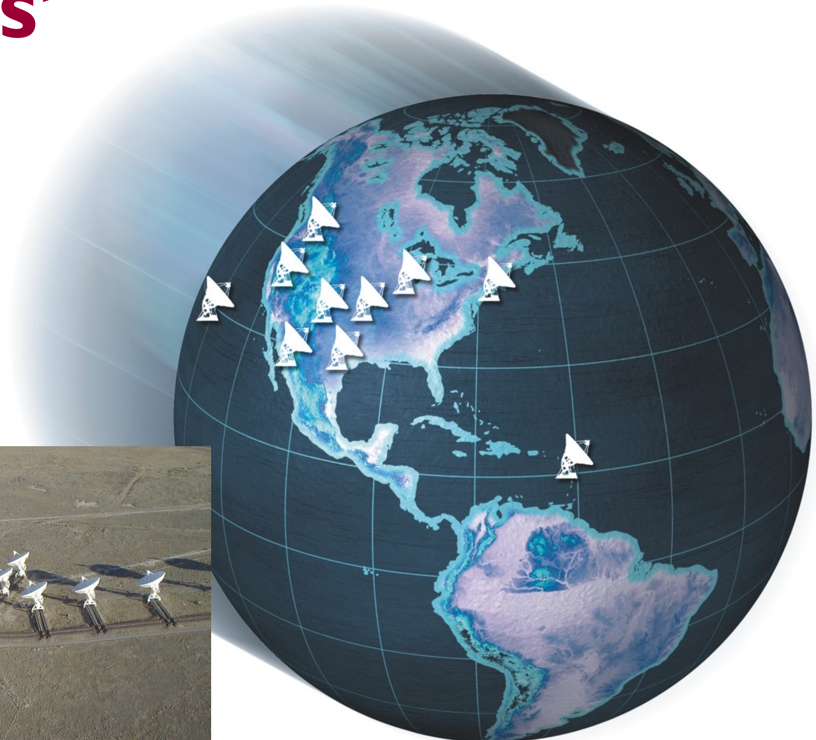
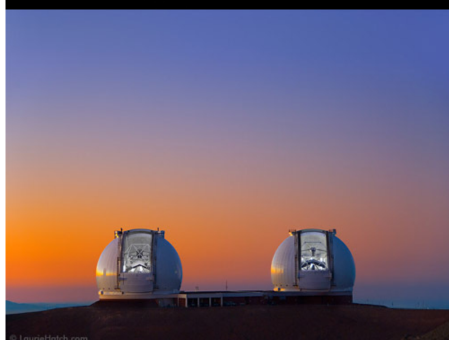


Image-forming ‘apertures’







These are apertures, too!

NRAO

What is focus?

- Curved lens w/ refractive index or curved mirror
  - Ensures a fixed distance from an isolated light source to the image point of that source in the image plane over all paths through the aperture.
  - More elegantly, this is a form of leveraging the least-action principle.
  - ➢ Mechanical isolation of the direction of arrival across an aperture to points in the image plane, thereby forming a crisp image
- Directly focused imaging requires mechanically pointing the aperture toward the direction of interest
  - NB: Focus deteriorates away from pointing direction: aberrations
  - Very expensive for large apertures (steel, glass, rockets…)
  - Impractical (or very limited) for radio astronomy at meaningful resolution
-

Resolution

- Larger apertures (as a multiple of wavelength) not only collect more light, they better isolate the direction of arrival

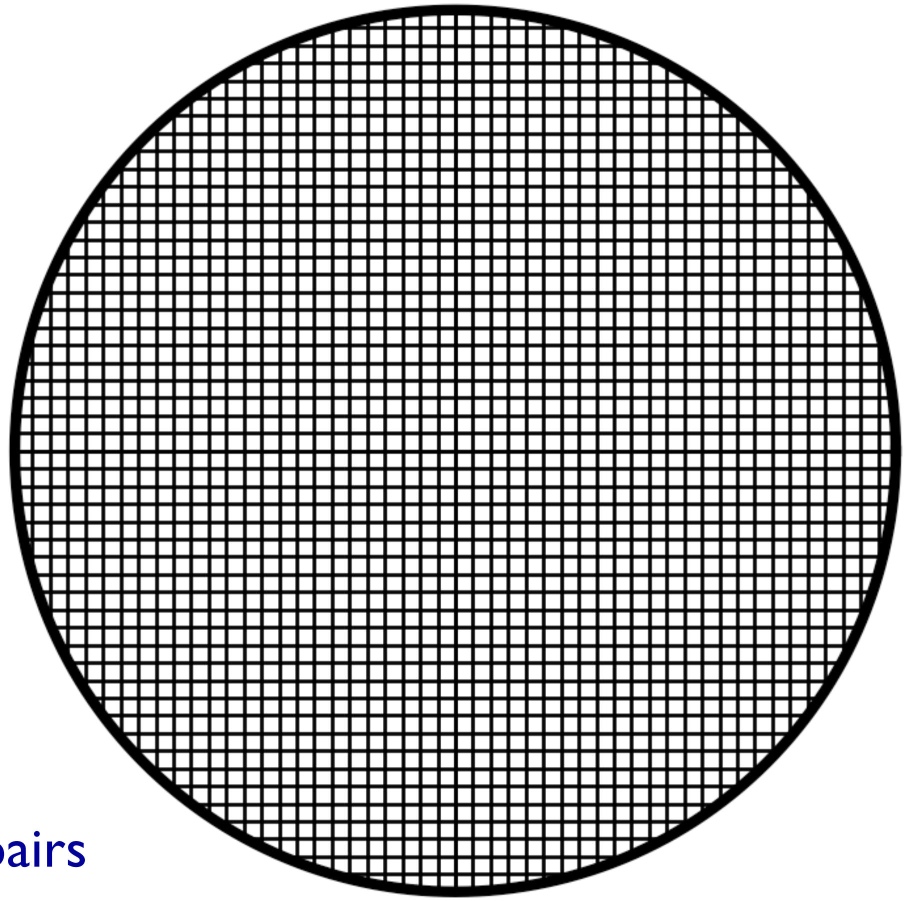
$$\theta_R \sim \lambda / D$$

- Optical: 650nm/10m ~ 0.013"
- Radio: 1cm / 10m ~ 200"
  - To achieve comparable resolution at radio and optical, radio telescopes (conventionally conceived) need to be 10000s times larger than optical telescopes! Impractical!
  - Modulo the relevant emission mechanisms, radio sensitivity with single apertures remains competitive (in terms of collecting area); just can't locate direction of arrival as well---and astrophysics demands better
- Must *synthesize* an equivalent aperture at radio wavelengths

- We shall unpack the properties of a light interacting with an aperture, and discover why this is so, ***and thereby how synthesis interferometry works!***

Segmented apertures

- Apparently, resolution arises from the *superposition* of light arriving from discrete sources across large apertures
  - So consider a segmented aperture…



  - For n segments, n(n-1)/2 distinct pairs

Segmented apertures; field vs. power

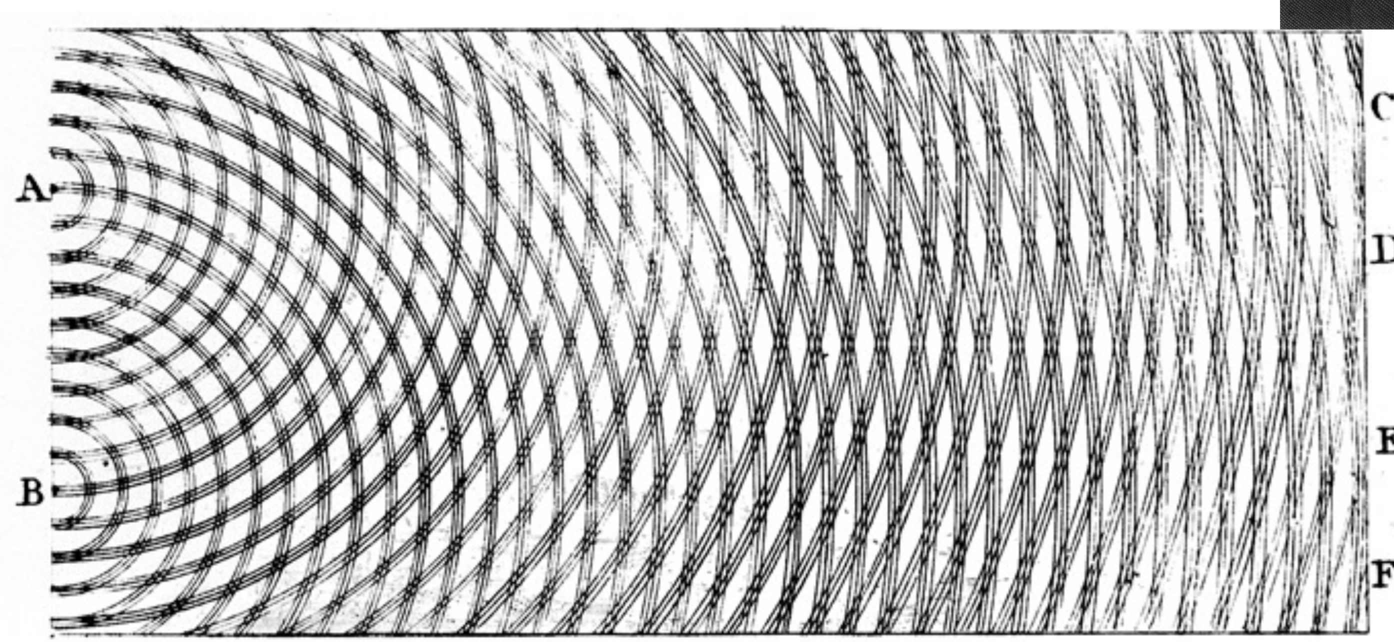
- The EM ***field*** at focus (for each discrete source) is a sum from all segments, *i*
  - Each segment would form the same (poorer) image by itself
- The image, a measurement of arriving EM ***power***, is the mean square of the net EM ***field*** arriving at each image pixel *(l,m)*

$$I(l,m) \sim \left\langle \left[\sum_{i} \sum_{l,m} s_i(l,m) \right]^2 \right\rangle$$

  - $s_i(l,m)$ is a (time- and frequency-dependent) signal arriving at segment *i*, as a function of direction
  - Light from all directions passes through each aperture segment…
  - …and yet each discrete source focuses separately and reliably on the image, somehow….
  - Self- and cross-power terms in the sum-of-squares
  - *Consider a single pair of aperture segments, on a baseline*

Young's Interference Experiment (~1804)

- Thomas Young (1773-1829)
- Demonstrated wave-like nature of light
  - Color corresponds to wavelength
- "Fringes" (and color!)
- Standard demonstration: ~monochromatic (laser)

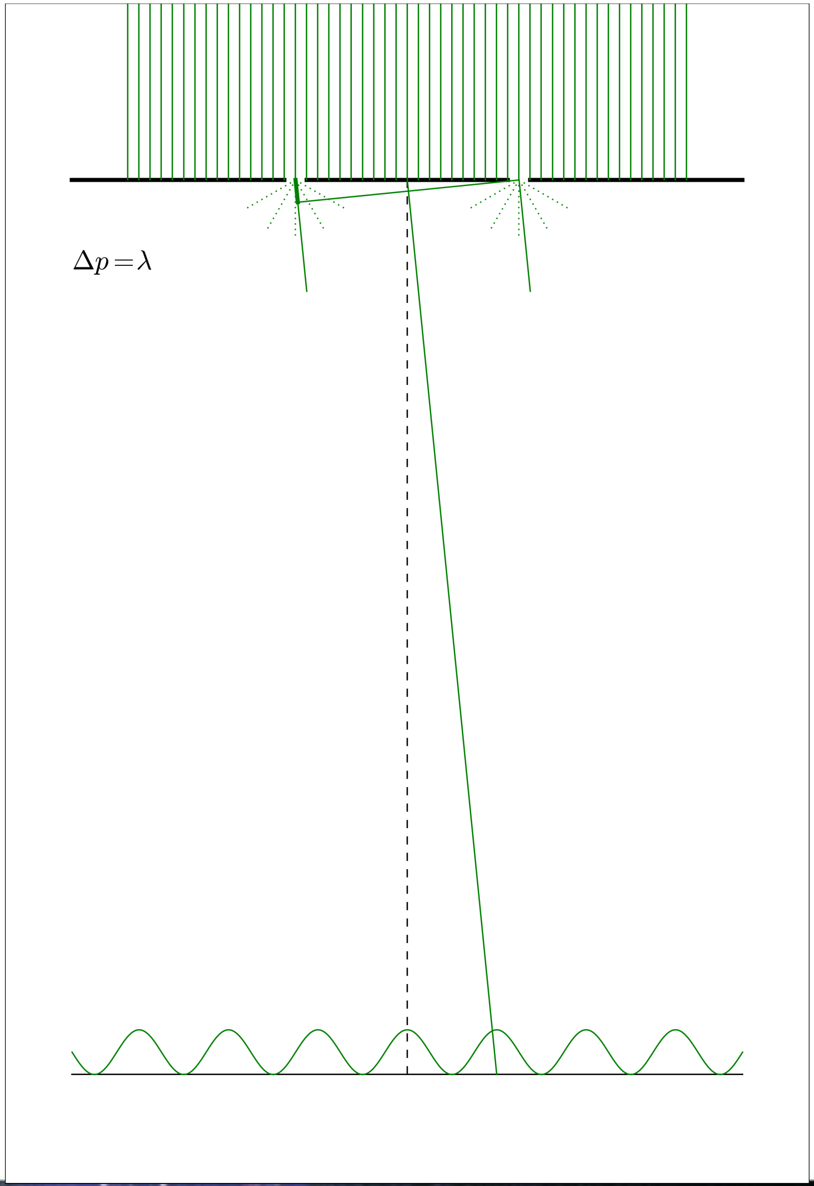


Constructive Interference

- For baseline length, *B*, and wavelength, λ, fringe **maxima** occur at angles, θ, such that (*n* any integer):

$$B\sin\theta = n\lambda$$

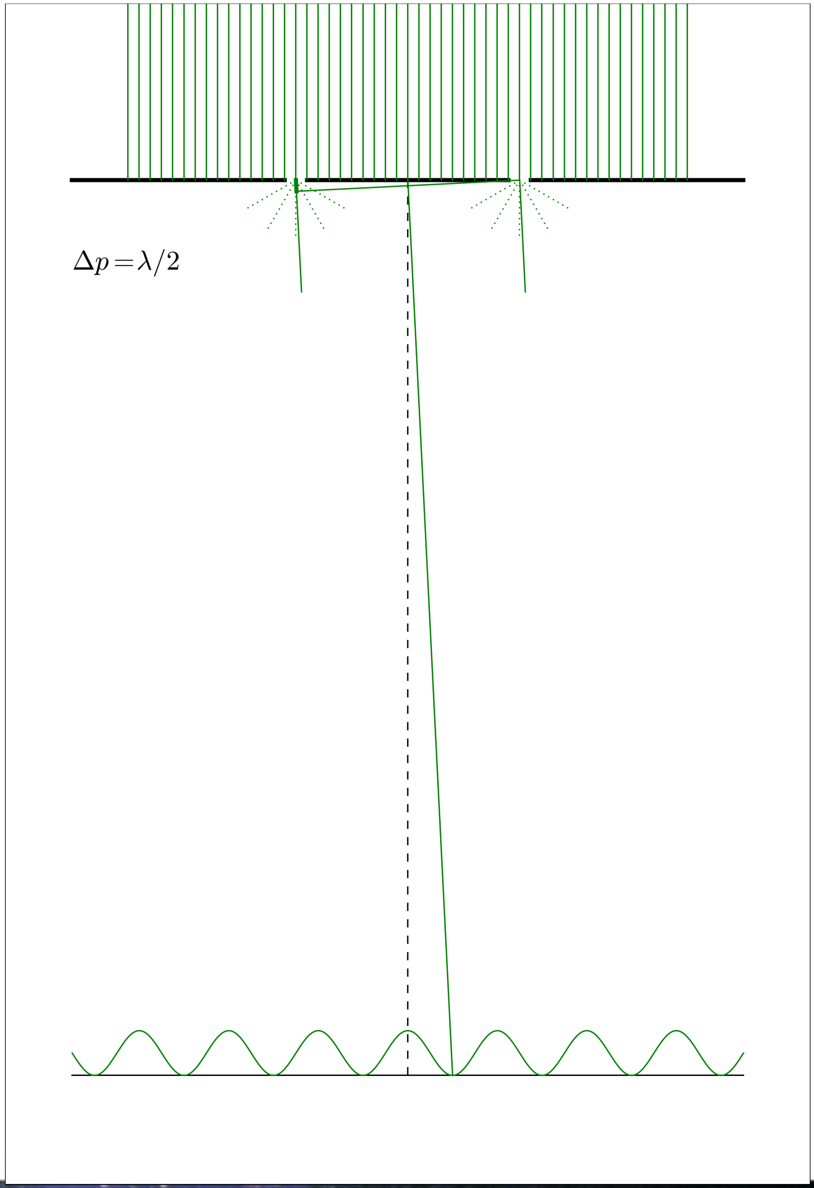
- Wider fringes for larger λ



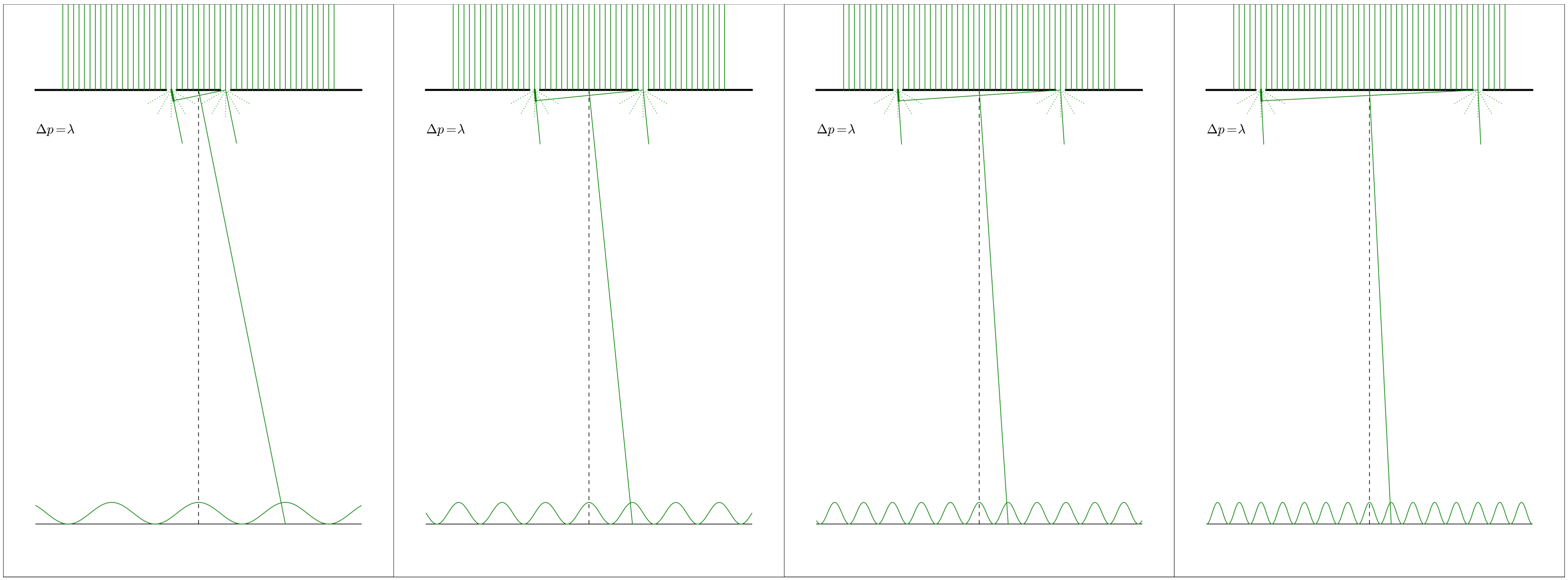
Destructive Interference

- For baseline length, *B*, and wavelength, λ, fringe **minima** occur at angles, θ, such that (*n* any integer):

$$B\sin\theta = \left(n+\tfrac{1}{2}\right)\lambda$$

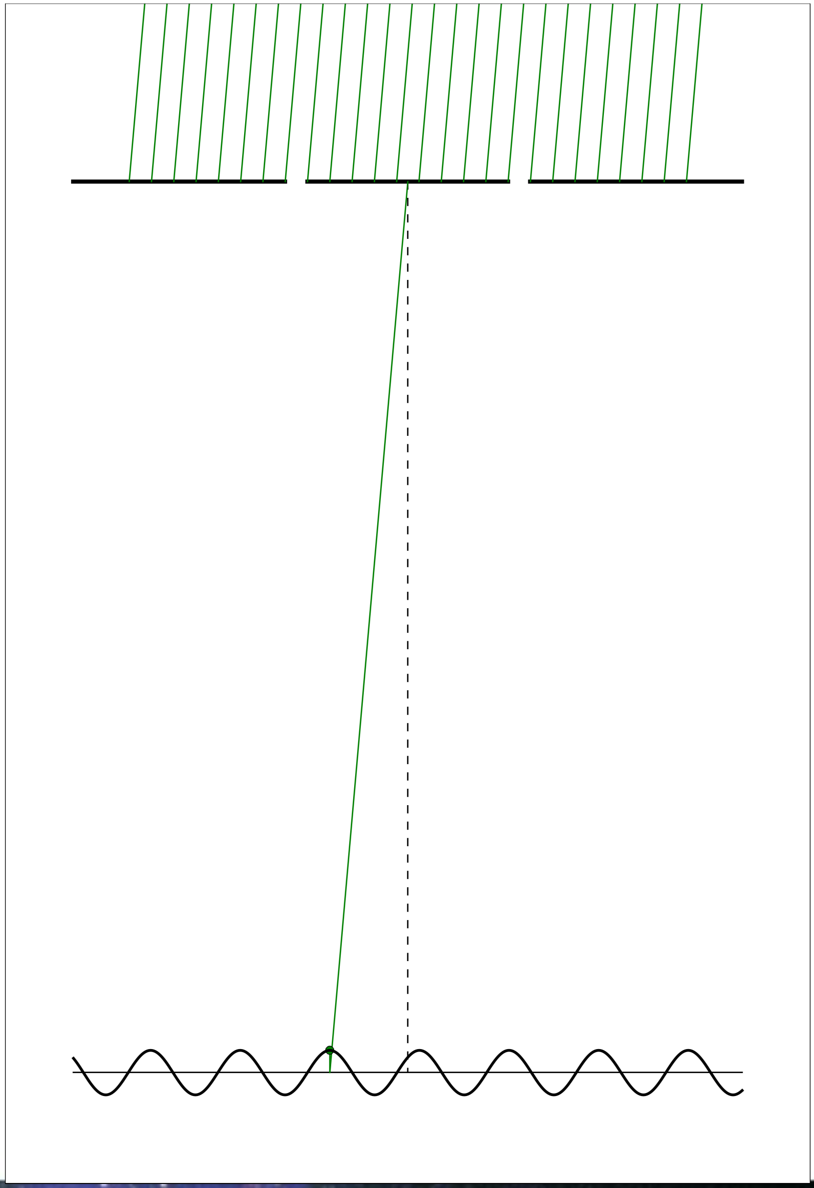


Increasing baseline length → Decreasing fringe spacing



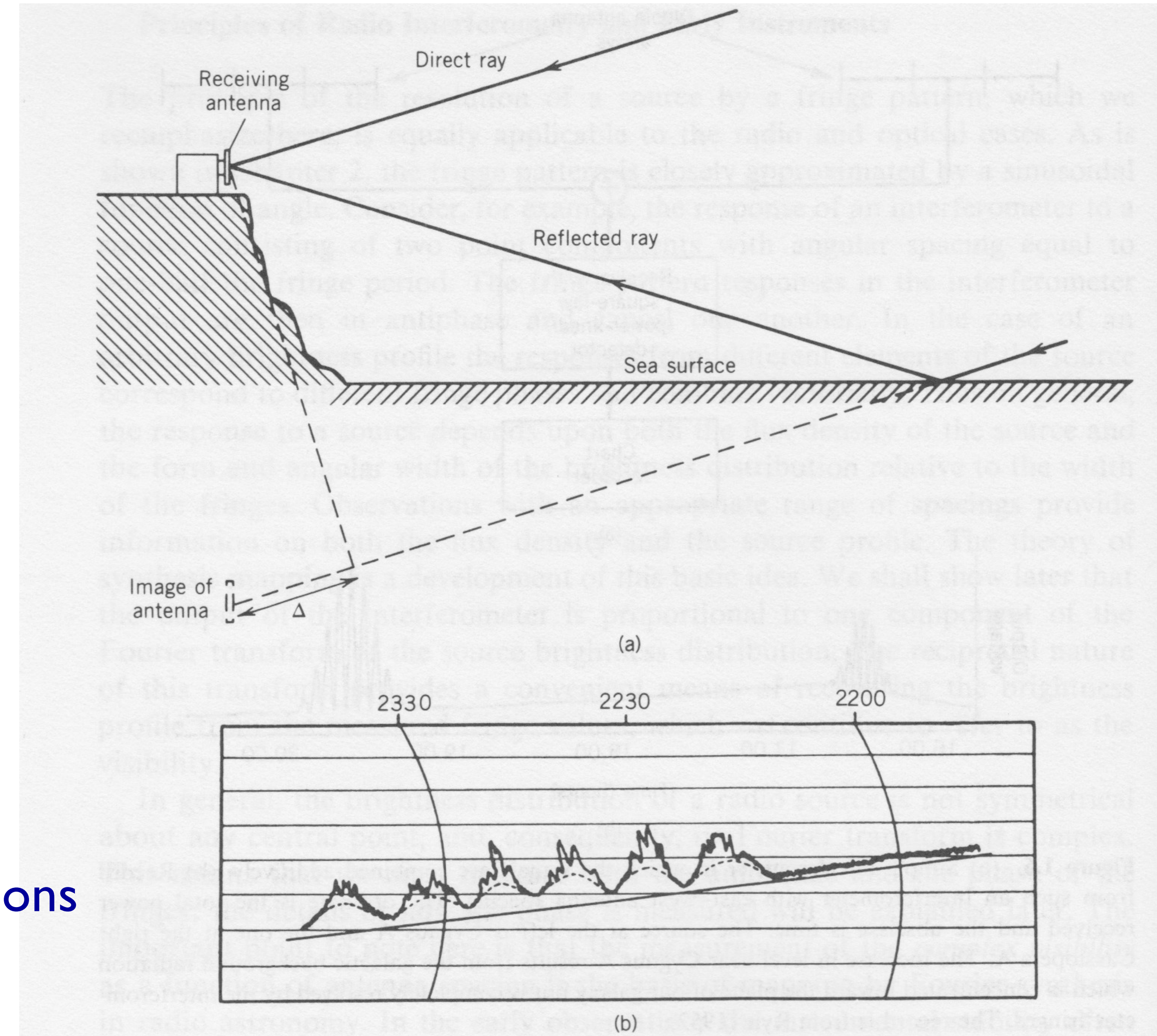
Generalized light source: off-axis

- Off-axis source moves central fringe maximum off-axis
- Fringe represents a very modest isolation of power in the image (by factor of 2)
  - Directional ambiguity



Sea Interferometer

- Bolton & Stanley (1948)



- Cygnus A at 100 MHz
  - Rising over the sea
  - Unresolved (<1°)
  - Ionospheric fluctuations

Fringe Power in Young's Interference

- Consider a (monochromatic) sinusoidal EM field disturbance from a single source arriving, via slits *i* and *j*, at a point on the screen where they add, differing only by a phase, ϕ, given by the geometry

$$s_i = E_0 \sin\left(\frac{2\pi ct}{\lambda}\right)$$
$$s_j = E_0 \sin\left(\frac{2\pi ct}{\lambda} - \phi\right)$$
$$s = s_i + s_j$$

- The measured power is proportional to the time-averaged square of the field:

$$I \propto \langle s^2 \rangle$$
$$= \left\langle \left(s_i + s_j\right)^2 \right\rangle$$

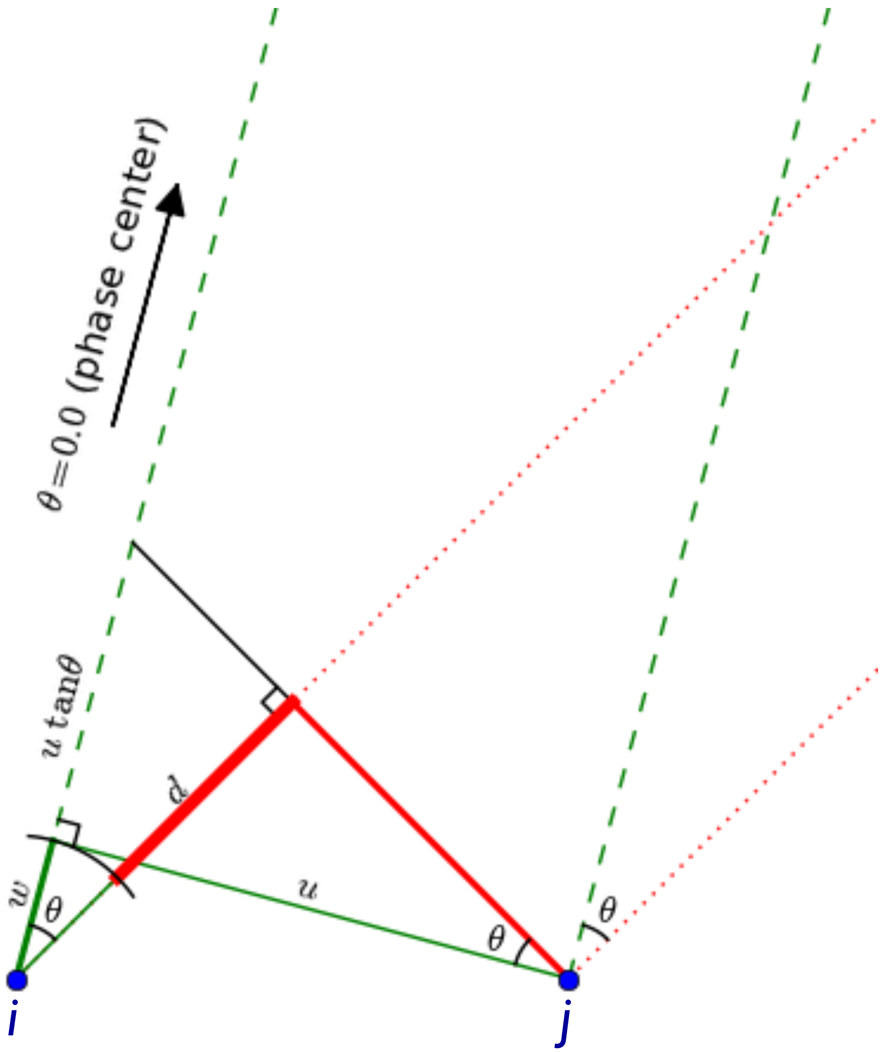
- Resolution information arises from the *cross-power* $\cos\phi$ term which generates the interference fringe

$$= \langle s_i^2 \rangle + \langle s_j^2 \rangle + 2\langle s_i s_j \rangle$$

- The *self-power* term contributes a *geometry-insensitive* offset (ensures non-negative power in a projected image)
  - We exclude the self-power terms below

$$= E_0^2 (1 + \cos\phi)$$

ϕ and signal arrival geometry, in general



$$\phi = 2\pi d(\theta)/\lambda$$

- Consider direction-dependent arrival geometry for E-field disturbance reception at two points, *i* and *j*, *relative to the phase center direction* (1D sky)
  - (E-field disturbance from each source arrives at both *i* and *j*.)

$$d = (w + u\tan\theta)\cos\theta - w$$
$$= u\sin\theta + w(\cos\theta - 1)$$
$$d(l) = ul + w\left(\sqrt{1-l^2} - 1\right)$$
$$\left(\sin\theta = l;\ \cos\theta = \sqrt{1-l^2}\right)$$

  - (SIN projection)

- Position-dependence of *w-compensated* direction-dependent signal:

$$s_j(l) = s_i(l)e^{i2\pi d(l)/\lambda}$$

Generalize for 2D sky, 3D receptor distribution

- Sky coordinates: *(l,m)*
- Relative segment coordinates: *(u,v,w)*

$$d(l,m) = ul + vm + w\left(\sqrt{1 - l^2 - m^2} - 1\right)$$
$$\approx ul + vm \qquad \left(|l| << 1, |m| << 1\right)$$

- Segment position dependence of the direction-dependent (w-compensated) signal:

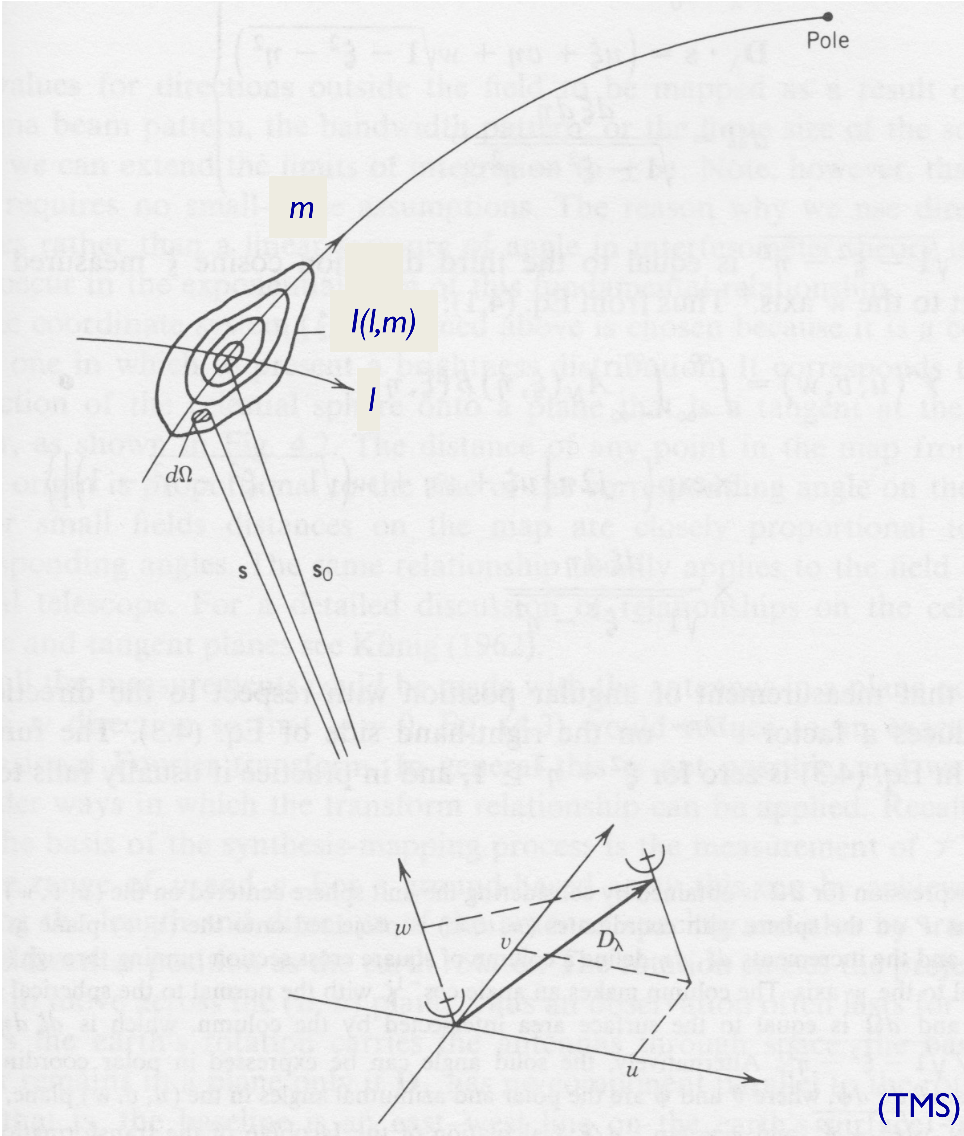
$$s_j(l,m) = s_i(l,m)e^{i2\pi d(l,m)/\lambda}$$

  - ***NB: absolutely no formal constraint on locations of aperture segments (e.g., need not be in a plane, etc.)***

Coordinates

- *(u,v,w)* are 3D baseline coordinates
  - w always points to the phase center
  - In units of the wavelength, *(u,v)* are basline projections along the on-sky coordinates

- *(l,m)* are 2D on-sky coordinates (SIN projection) relative to the phase center *(l=m=0)*



(TMS)

Formation of the visibility: correlation

- Correlation of sky-aggregate w-compensated signals received at aperture segments *i* and *j*:

$$V_{ij} = \left\langle \int_{sky} s_i(l,m)dldm \cdot \int_{sky} s_j^*(l,m)dldm \right\rangle_{\Delta t}$$

- Sky integral and signal product can be reversed, because the E-field disturbances from different directions don't correlate
  - *for finite bandwidth*

$$V_{ij} = \left\langle \int_{sky} s_i s_j^* dldm \right\rangle_{\Delta t}$$

- Substitute for s_j and reverse the sky integral and time average

$$V_{ij} = \int_{sky} \langle |s_i|^2 \rangle e^{-i2\pi d(l,m)/\lambda} dldm$$

- The mean square direction-dependent signal $\langle s_i \rangle$ is just the brightness distribution:

$$V_{ij} = \int_{sky} I(l,m) e^{-i2\pi d(l,m)/\lambda} dldm$$

Mutual incoherence of natural light sources

- That different natural sources of light are mutually incoherent makes image formation possible!
  - All EM fields superpose at each segment
  - In correlation, only *per-source* fringe power terms survive, and these are superposed in the visibility, forming a single aggregate fringe
    - Broad ~uniform brightness distributions yield weak visibilities
- Requires finite bandwidth per correlation measurement
  - Truly monochromatic light sources would necessarily be coherent at some arbitrary phase
    - C.f., modern Young's interference formalism
  - Integral over a finite bandwidth of monochromatic correlations with randomly-distributed phase tends to zero
  - "Quasi-monochromatic approximation"
  - All of the above analysis occurs over a small but distinctly finite bandwidth centered at λ

The Visibility Function

- *Exact* general expression (to within power scale factor):

$$V(u,v,w) = \int_{sky} I(l,m) e^{-i2\pi\left[ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right]/\lambda} \, dl \, dm$$

  – *w* term represents off-axis aberration (c.f. conventional optics)
- For small fields of view (e.g., VLBI, especially!), w-term is ~zero:

$$V(u,v) = \int_{sky} I(l,m) e^{-i2\pi(u_\lambda l + v_\lambda m)} \, dl \, dm$$

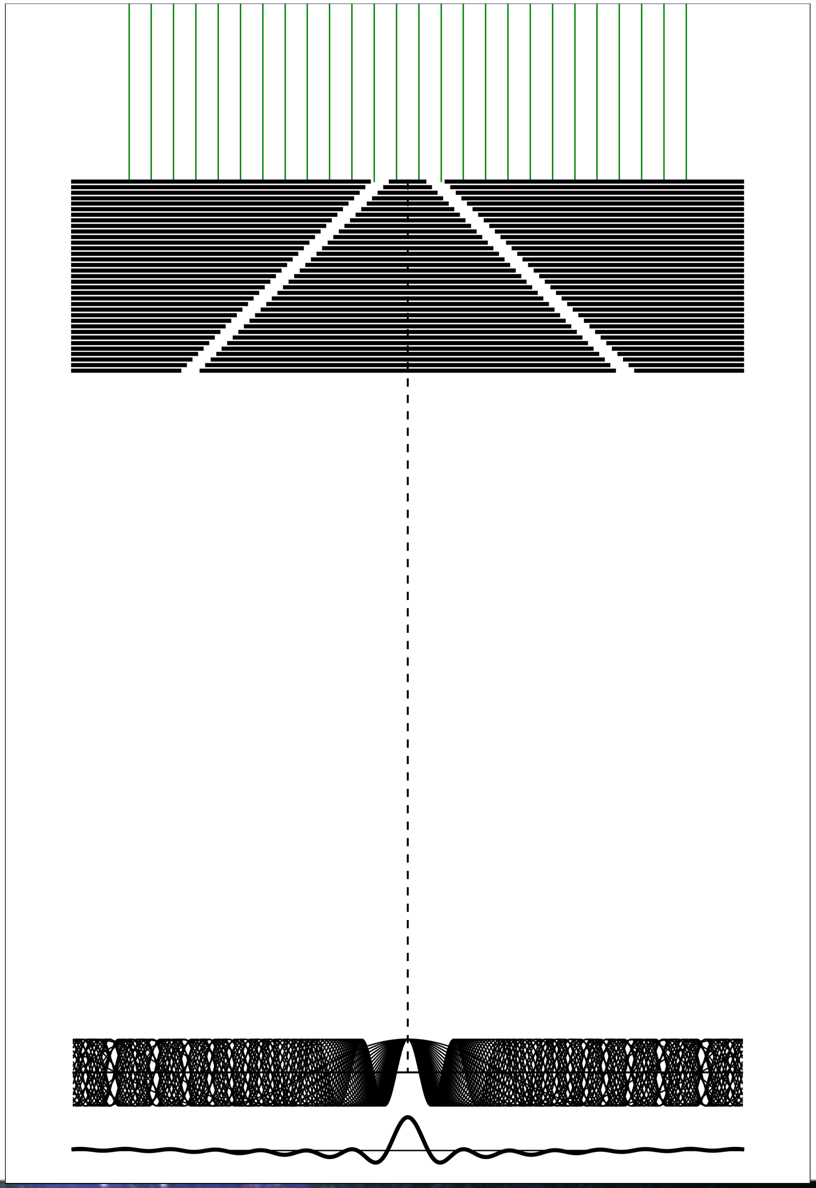
  – Van Cittert-Zernike Theorem: ***The visibility function is the 2D Fourier transform of the brightness distribution***
  – Measure $V(u,v)$ as completely as possible, and merely invert to form the image!
  – We have synthesized an aperture via electronics and computation!
  – Sparse sampling: ugly point-spread function ("diffraction spikes")
    - Imaging and deconvolution (Christiana on Wednesday)

NRAO

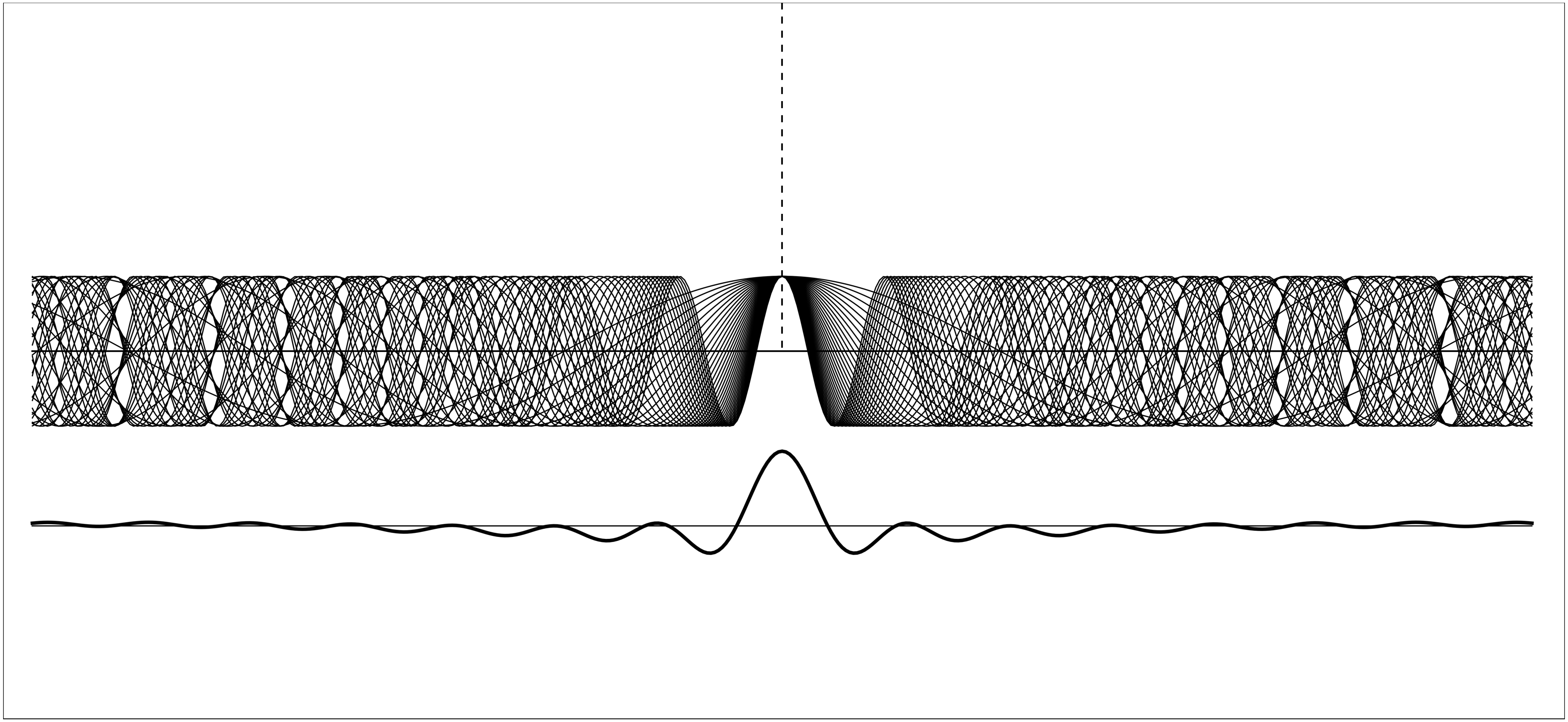
Superposition of Fringes (Power)

- 36 baselines (5:6: … :40)
- Single point source
- Imperfect point-spread function (psf)

- Conventional filled aperture is just the sum of a more complete sample of fringes
  - (weighted more towards shorter spacings, which are more numerous)

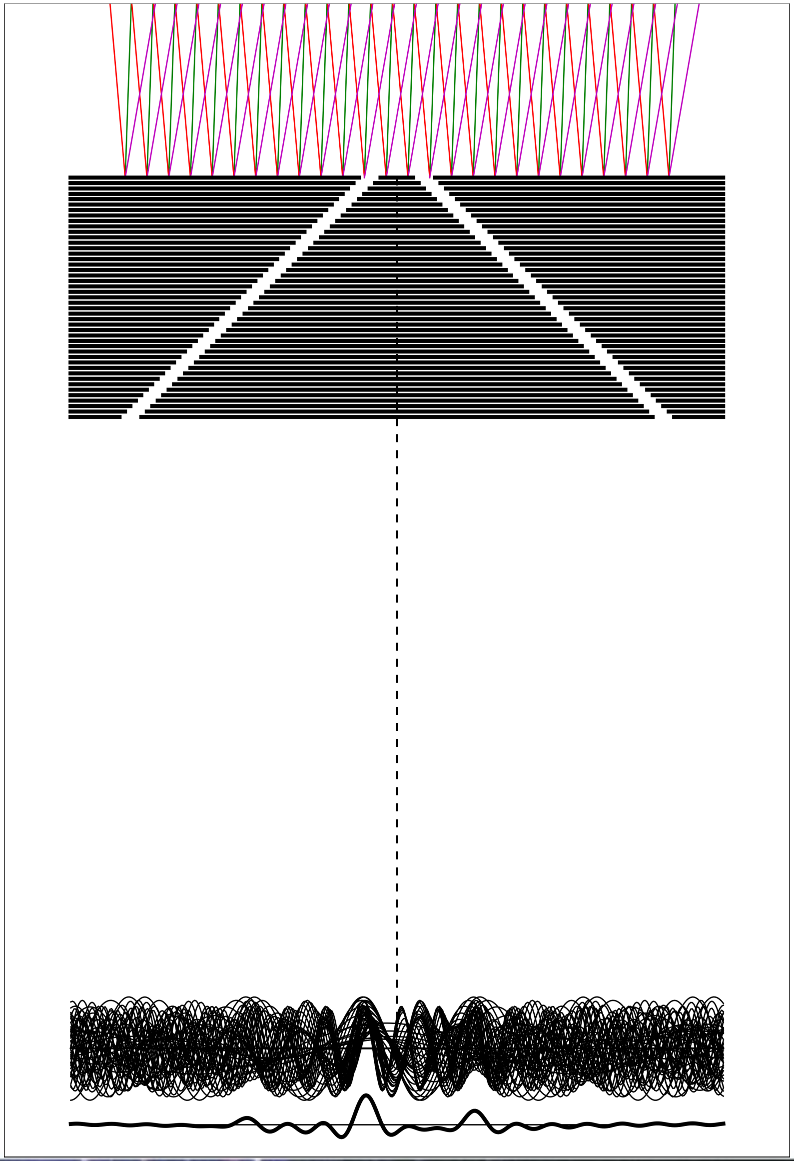


Superposition of Fringes (Power)

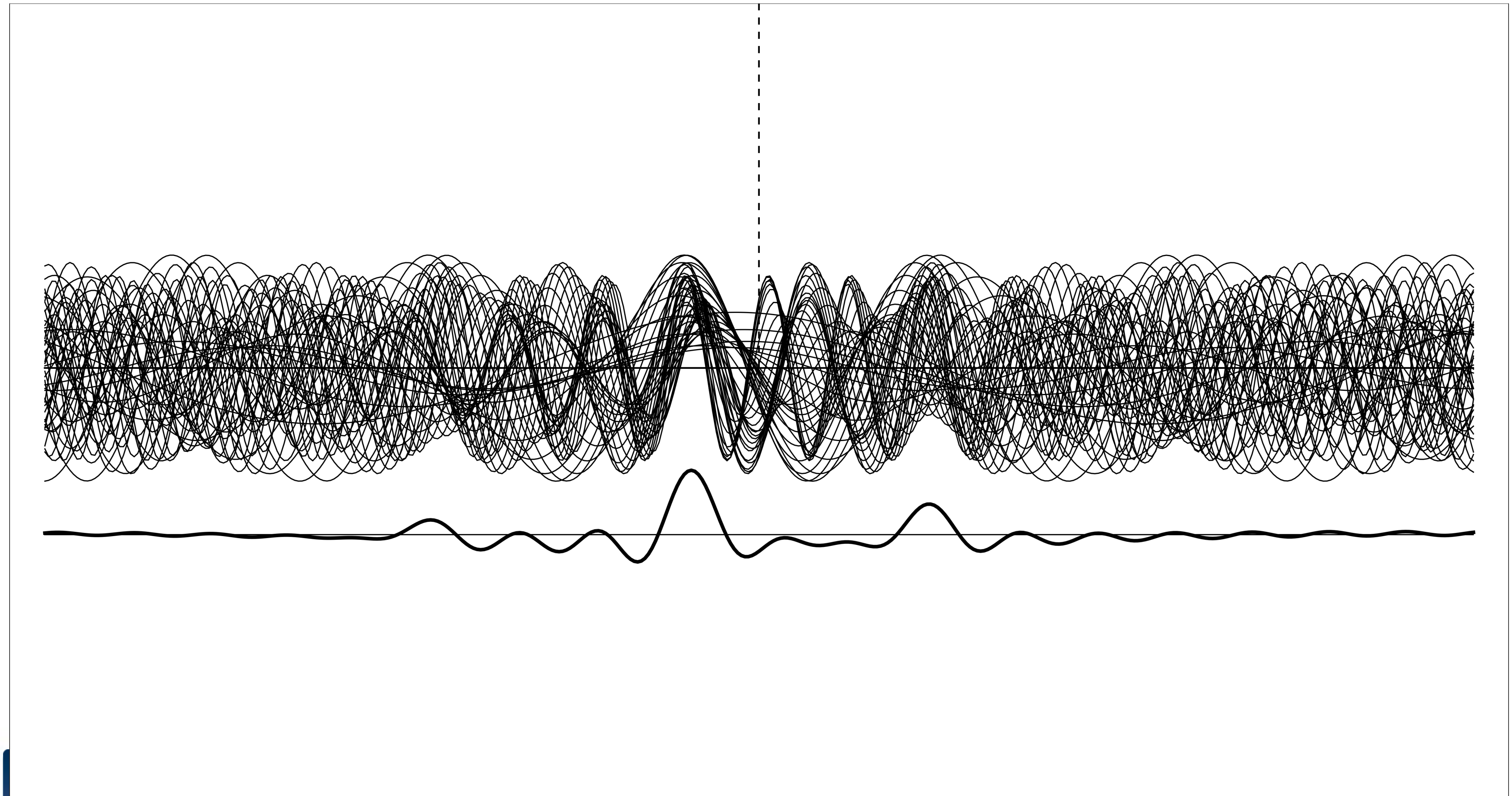


Generalized light source

- Discrete sources in three different directions
- Note apparent convolution by the psf



Generalized light source: multiple

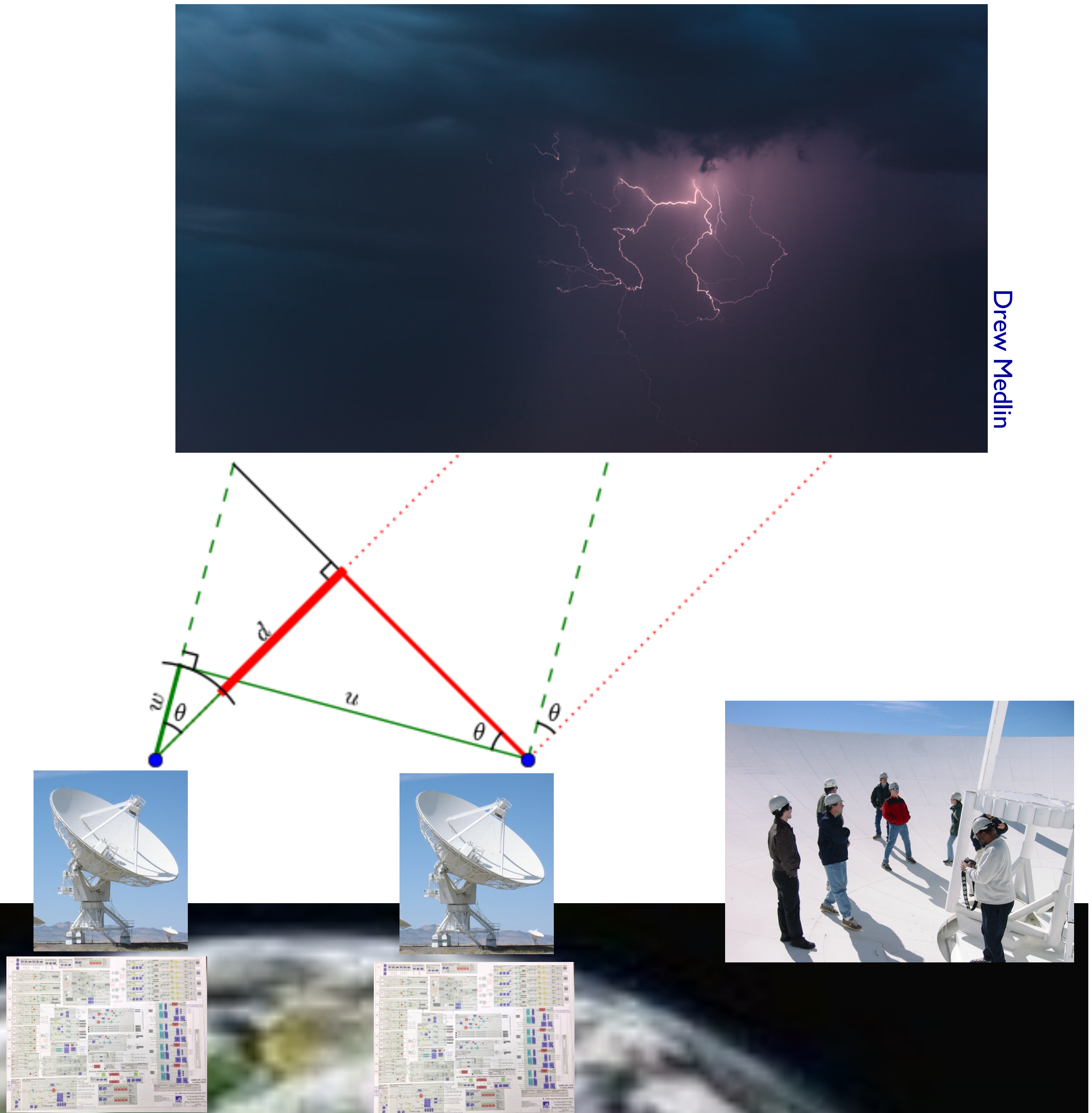


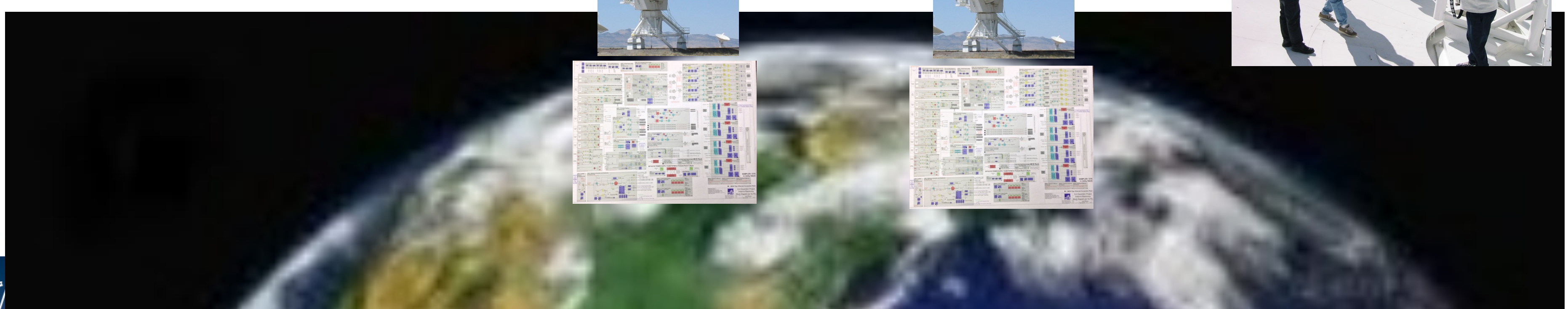
Technical details

- Practical realization of a synthesis interferometry requires a lot of finite (i.e., imperfect) mechanical and electrical *engineering*
  - Antenna design/operation
  - Down-conversion
  - Level-setting, quantized sampling
  - Signal transmission (including via FedEx for VLBI!)
  - Delay- and phase-tracking (moving platform)
  - Correlation

Reality can be messy…

- Weather
- Realistic antennas
- Electronics…
- Digital correlation
- …and the whole thing is moving!





Why Calibration?

- Synthesis radio telescopes, though well-designed, are not perfect (e.g., surface accuracy, receiver noise, polarization purity, gain stability, geometric model errors, etc.)
- Need to accommodate deliberate engineering (e.g., frequency down-conversion, analog/digital electronics, filter bandpass, etc.)
- Hardware or control software occasionally fails or behaves unpredictably
- Scheduling/observation errors sometimes occur (e.g., wrong source positions)
- Atmospheric conditions not ideal
- Radio Frequency Interference (RFI)

Determining *instrumental and environmental properties* (calibration)
is a prerequisite to
determining *radio source properties*

NRAO

Antenna-based calibration

- Each aperture segment is a fundamental observational element about which we have imperfect real-time knowledge of its condition: location, performance, weather, etc.
- The signal delivered by each element is thus corrupted

$$s_i(t, \nu, l, m) \rightarrow J_i(t, \nu, l, m)\, s_i(t, \nu, l, m)$$

  - $J_i(t, \nu, l, m)$ is a complex number characterizing the full range of effects encountered by s_i in its flight toward and through aperture segment *i*
  - J_i (and/or combination with others in pair-wise signal correlation) may sometimes corrupt data irrevocably, and the corresponding data must be *flagged*

J_i and Computational Optics

- Does J_i compromise our computational optics?
  - No! As long as we sample our visibility measurements at a granularity adequate to adequately describe J_i (→ big datasets!)
  - The J_i factors just follow their corresponding signals around…

$$V_{ij}^{obs} = \int_{sky} J_i(t,\nu,l,m)\, J_j^*(t,\nu,l,m)\, I(l,m) e^{-i2\pi\left(u_{ij}l+v_{ij}m\right)d(l,m)/\lambda} dldm$$

  - We can factor direction-dependent components and remove direction-independent J_i from the integral

$$V_{ij}^{obs} = J_i J_j^* \int_{sky} J_i^{dd}\, J_j^{dd*}\, I(l,m) e^{-i2\pi\left(u_{ij}l+v_{ij}m\right)d(l,m)/\lambda} dldm$$

Visibility calibration

- Over small fields of view (e.g., VLBI!), we can assume $J_i^{dd} = 1.0$ and write:

$$V_{ij}^{obs} = J_i J_j^* V_{ij}$$

- Standard calibration reduces to solving this last equation for the complex J_i assuming a visibility model V_{ij}^{mod}
  - Ideally, using periodic observations of a point-like calibrator ("phase-referencing")
  - Ideal calibrators not always available for VLBI → iterative self-calibration (Javier on Wednesday)

- Insofar as antenna-based visibility calibration accounts for errors in the effective aperture, it amounts to performing adaptive optics in post-processing

NRAO

Calibration factorization

- In practice, it is convenient to factor J_i into separate terms corresponding to different elements and aspects of the signal path:

$$J_i = K_i B_i G_i D_i P_i T_i F_i$$

- Where:
  - K_i = geometry, timing errors and similar
  - B_i = frequency-dependent complex gain
  - G_i = general complex gain calibration, including amplitude calibration
  - D_i = instrumental polarization calibration
  - P_i = parallactic angle: signal polarimetric orientation
  - T_i = tropospheric path
  - F_i = ionospheric calibration
- Because EM wave is a vector, these calibration factors are matrices, and order matters (software handles all of that)
- This factorization is approximate; some effects bleed together
  - Calibration heuristics…

Amplitude calibration

- Convert visibility power to scientifically useful units, via antenna-based amplitude factors, $|G_i|$
  - Correction of residual quantization scaling errors
    - CASA task: accor
  - Tsys (K) calibration, to compensate for effective normalization implicit in correlation of quantized EM field samples
    - CASA task: gencal
  - Aperture efficiency to convert K to Jy
    - CASA task: gencal

- Mark on Tuesday

Fringe-fitting

- Solve and correct for timing and geometry errors, K_i, in correlator's apprehension of *w(t)* (including atmospheric effects arising in T_i *and* F_i), the antenna-based signal delays required to establish the correlation phase center. Solved via modeling the nominal visibility phase as a function of time and frequency, parameterized by delay, delay-rate, and dispersive delay (CASA task: fringefit)
  - Particularly relevant to VLBI, where antenna clocks are independent, and relative antenna locations difficult to maintain at accuracy comparable to wavelength

- Des on Tuesday

Bandpass calibration

- Solve and correct for non-linear frequency-dependent complex gain
  - CASA task: bandpass

- Olga on Wednesday

Gain (self-)calibration

- Solve and correct for residual complex gain (amp and phase), after gross errors have been removed by prior calibration, possibly in an iterative manner with an alternately improved source visibility model (self-cal)
  - CASA task: gaincal

- Javier on Wednesday

Polarization calibration

- Solve and correct for imperfect response, D_i, in dual polarization feeds, wherein each feed sees a small fraction of the unintended polarization. Also parallactic angle, P_i
  - CASA task: polcal

- Ivan on Thursday

Generalizations

- All of this interference phenomena occurs implicitly for conventional single apertures–*diffraction*
  - Conventional apertures have missing aperture sampling, too, just much less (diffraction spikes); most of the geometry calibration occurs at construction
- Consider: JWST panel-setting
  - Coarse: 18 distinct (low-resolution) images, superposed in *power*
  - Fine: coherent superposition of EM *field* before detecting power
- Consider: Adaptive optics (AO) on optical telescopes: real-time fine-tuning of focus across an aperture
- Consider: Optimizing the performance of each antenna within a synthesis array: surface holography and panel-setting
- Consider: forming phased-array elements for VLBI arrays: AO for a uniquely-segmented aperture

These are all variations on the same theme

Summary

- Image formation from EM radiation can be intuitively understood via a very generalized concept of segmented apertures
- Radio astronomy achieves high resolution despite long wavelength via electronic/computational optics
  - Limited only by how far apart aperture segments can be placed and timing/geometry adequately maintained (some degree of recovery possible in calibration)
- Mutual incoherence of discrete radiation sources required for sensible image formation
- Calibration required to refine geometry and relative response among aperture segments
  - Adequate sampling for calibration can yield large datasets
- In principle, any finite aperture has finite resolving power and can be analyzed as above; this is a fundamentally sound expression of generalized diffraction

Questions?

- *"This event has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 101004719"*

www.nrao.edu
science.nrao.edu

The National Radio Astronomy Observatory is a facility of the National Science Foundation operated under cooperative agreement by Associated Universities, Inc.