Systematic effects in LOFAR

CHRISTIAN GROENEVELD, LEIDEN OBSERVATORY

Overview

Jones Formalism

Calibration fundamentals

Overview of systematic effects

Tutorial

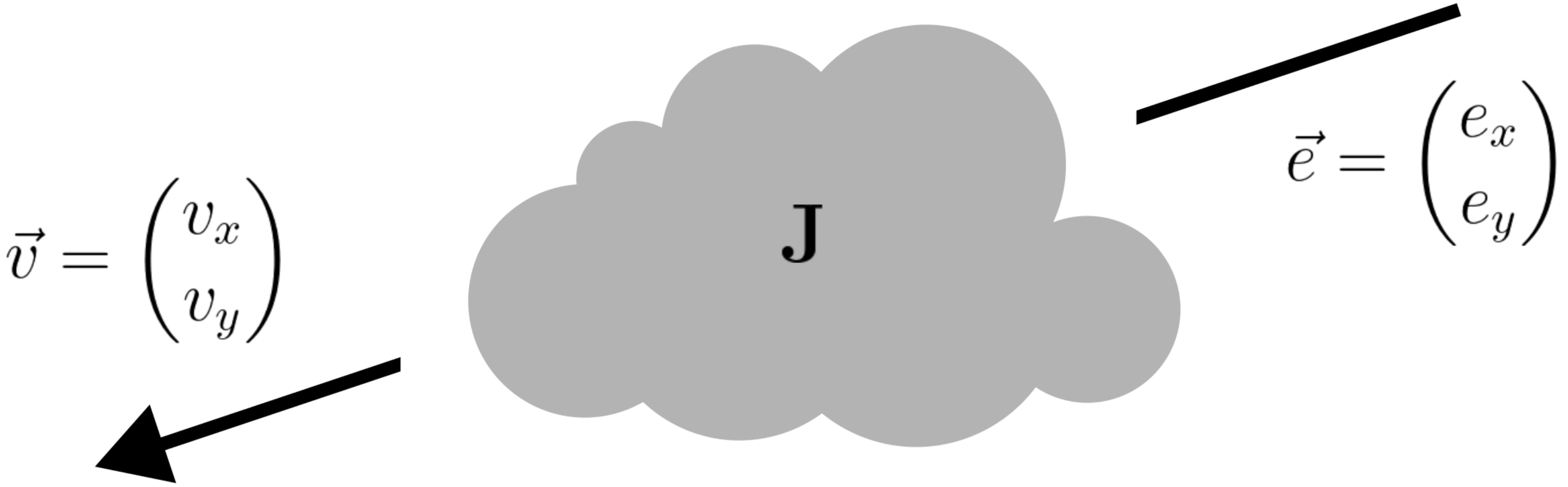
Linking signal to voltage

Detector: Voltage vector

$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

Radio source: Signal

$$\vec{e} = \begin{pmatrix} e_x \\ e_y \end{pmatrix}$$



Linked by series of matrices: Jones matrix

- Each effect has corresponding matrix

$$\vec{v} = \mathbf{J}\vec{e}$$

Jones Matrices

From signal to voltage: chain of Jones matrices

Note: matrix multiplication in general not commutative!

$$\vec{v} = \mathbf{J}_1\mathbf{J}_2\mathbf{J}_3...\mathbf{J}_n\vec{e}$$
$$= \mathbf{J}\vec{e}$$

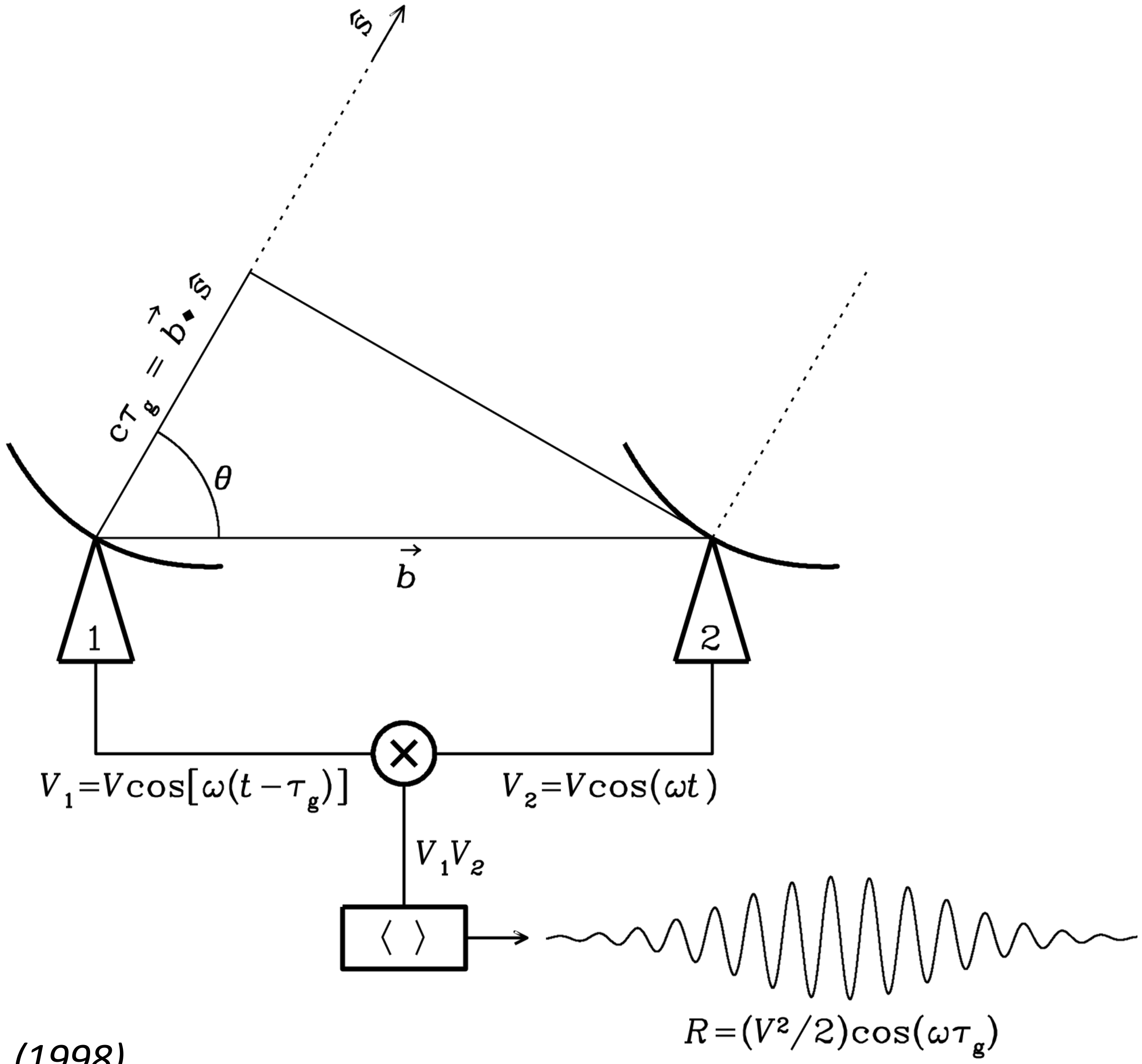
Each effect has it's own Jones matrix

Jones Formalism and visibilities (1)

Link between sky intensity distribution and mutual coherence function: **van Cittert-Zernike theorem** *(Thompson, Moran, Swenson // Oei)*

$$\left\langle \vec{E}(\vec{r_1})\vec{E}(\vec{r_2})^\dagger \right\rangle \approx \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_\nu(l,m)e^{-2\pi i(ul+vm)}\mathrm{d}l\mathrm{d}m$$

<u>Sky intensity distribution</u> and <u>mutual coherence function</u> are Fourier pairs!



Condon and Ransom (1998)

Jones Formalism and visibilities (2)

Construction of mutual coherence: Assume *quasi-monochromatic* interferometer

Then two interferometers have E-field:

$$V_1 = V\cos(\omega t)$$
$$V_2 = V\cos(\omega(t - \tau_g))$$
$$\tau_g = \frac{\vec{b}\cdot\hat{s}}{c}$$

Multiply, then take time average

(removes fast component)

$$\begin{aligned}\mathcal{R}_c &= \langle V_1 V_2^\dagger \rangle = \langle V^2 \cos(\omega t)\cos(\omega(t-\tau_g))\rangle \\ &= \left\langle \frac{V^2}{2}[\cos(2\omega t - \omega\tau_g) + \cos(\omega\tau_g)]\right\rangle \\ &= \frac{V^2}{2}\cos(\omega\tau_g)\end{aligned}$$

Jones Formalism and visibilities (3)

Do the same for sine-part of signal. **Complex Visibility:**

$$\mathcal{V} = \mathcal{R}_c + i\mathcal{R}_s = Ae^{-i\phi}$$

If you have two feeds, e.g. x and y-feed (polarization), you will get four visibilities per unique pair of antennas: $\mathcal{V}_{xx}, \mathcal{V}_{xy}, \mathcal{V}_{yx}, \mathcal{V}_{yy}$

Complex visibility can form a matrix:

$$\mathbf{V} = \langle \vec{v}_1 \vec{v}_2^\dagger \rangle = \begin{pmatrix} \mathcal{V}_{xx} & \mathcal{V}_{xy} \\ \mathcal{V}_{yx} & \mathcal{V}_{yy} \end{pmatrix}$$

Jones Formalism and visibilities (4)

How does Jones matrix play into this?

Recall: $\vec{v} = \mathbf{J}\vec{e}$

$$\mathbf{V}_{12} = \langle \vec{v}_1 \vec{v}_2^\dagger \rangle = \langle \mathbf{J}_1 \vec{e}_1 (\mathbf{J}_2 \vec{e}_2)^\dagger \rangle = \mathbf{J}_1 \langle \vec{e}_1 \vec{e}_2^\dagger \rangle \mathbf{J}_2^\dagger \quad , \quad \langle \vec{e}_1 \vec{e}_2^\dagger \rangle \equiv \mathbf{B}$$

N-element interferometer: N^2-N/2 visibilities, N Jones matrices

Radio interferometery measurement equation (RIME)

$$\boxed{\mathbf{V}_{ij} = \mathbf{J}_i \mathbf{B}_{ij} \mathbf{J}_j^\dagger}$$

Calibration

Goal of calibration: Find relevant Jones matrices

- Reconstruct B from measured V

Given: a starting model (reference visibilities)

Solve: $||\mathbf{V}_{\mathrm{obs},ij} - \mathbf{J}_i \mathbf{V}_{\mathrm{model},ij} \mathbf{J}_j^\dagger||$ for $\mathbf{J} = \begin{pmatrix} A_{xx}e^{i\phi_{xx}} & A_{xy}e^{i\phi_{xy}} \\ A_{yx}e^{i\phi_{yx}} & A_{yy}e^{i\phi_{yy}} \end{pmatrix}$

`DP3`

Strategies: talk by Reinout van Weeren

Calibrators

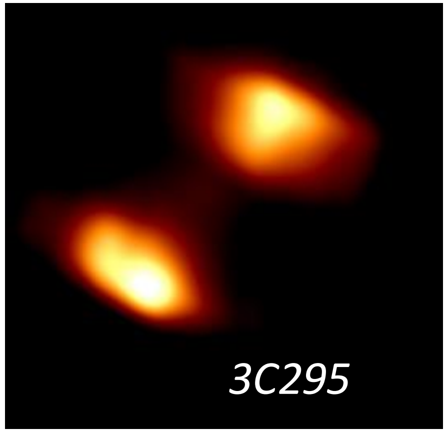
Reference object, we know the structure

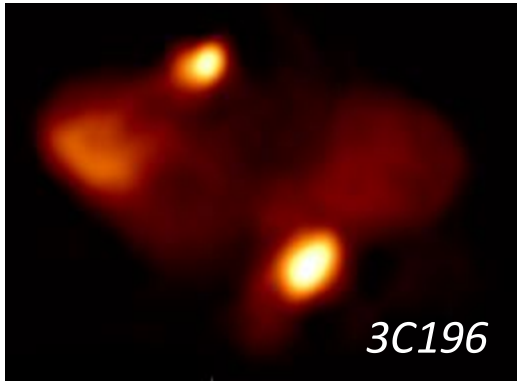
Very bright (high signal to noise)

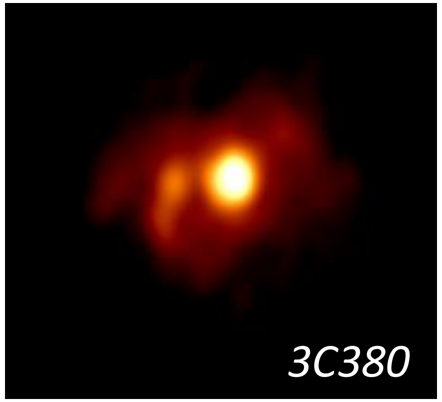
Compact emission

- Diffuse emission is missed by long baselines

Well defined spectrum (Scaife and Heald, 2012)







Solutions

Calibration generates solutions (complex number)

- Each solution *(gain)* consists of an amplitude and a phase

Solutions: for each time, frequency, polarization, calibration gives amplitude+phase part

Solutions contain both *Ionospheric* and *systematic* effects

Instrumental Systematic effects

Systematic effect	Type of Jones matrix[a]	Ph/Amp/Both[b]	Frequency dependency	Direction dependent?	Time dependent?
Clock drift	Scalar	Ph	$\propto \nu$	No	Yes (many seconds)
Polarisation alignment	Diagonal	Ph	$\propto \nu$	No	No
Ionosphere - 1st ord. (dispersive delay)	Scalar	Ph	$\propto \nu^{-1}$	Yes	Yes (few seconds)
Ionosphere - 2sn ord. (Faraday rotation)	Rotation	Both	$\propto \nu^{-2}$	Yes	Yes (few seconds)
Ionosphere - 3rd ord.	Scalar	Ph	$\propto \nu^{-3}$	Yes	Yes (few seconds)
Ionosphere - scintillations	Diagonal	Amp	–	Yes	Yes (few seconds)
Dipole beam	Full-Jones	Both	–	Yes	Yes (minutes)
Bandpass	Diagonal	Amp	–	No	No

De Gasperin et al. 2019

Clock drift

Remote stations have individual clocks, that slowly drift

- O(10ns/hr)

Introduces scalar phase error: $\Delta\phi = 2\pi\nu\Delta t$

Scales as $\sim \nu$

$$\mathbf{J}_{i,\mathrm{clock}} = e^{2\pi i\nu\Delta t}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

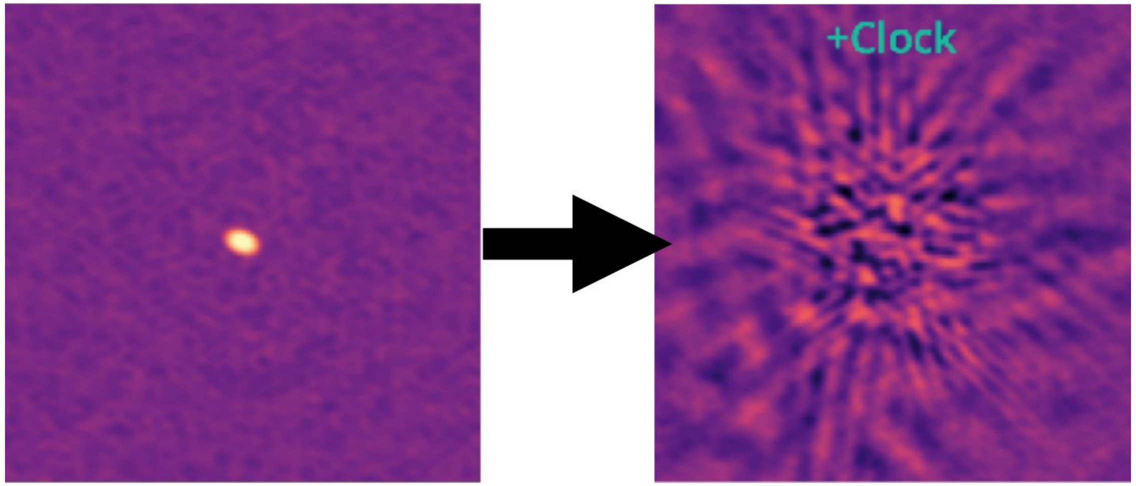


Image credit: Henrik Edler

Polarisation misalignment

Small time delay between X- and Y-feeds (on station basis)

$$\mathbf{J}_{i,\text{pol. align}} = \begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i \nu \Delta t} \end{pmatrix}$$

O(ns)

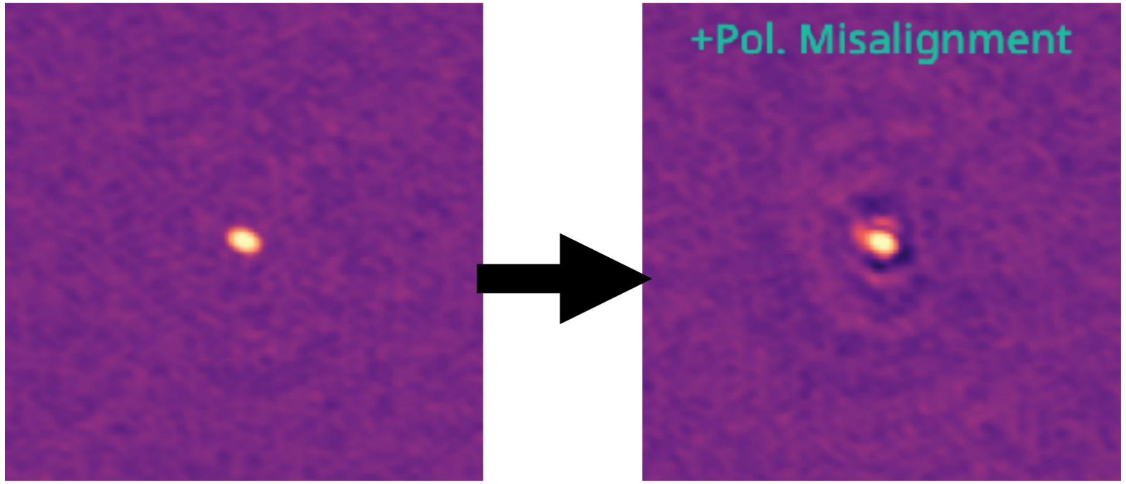


Image credit: Henrik Edler

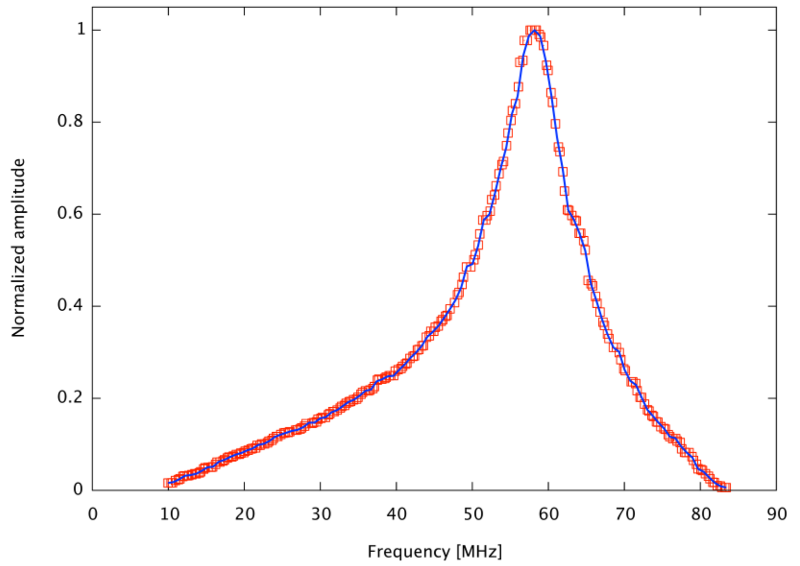
Bandpass

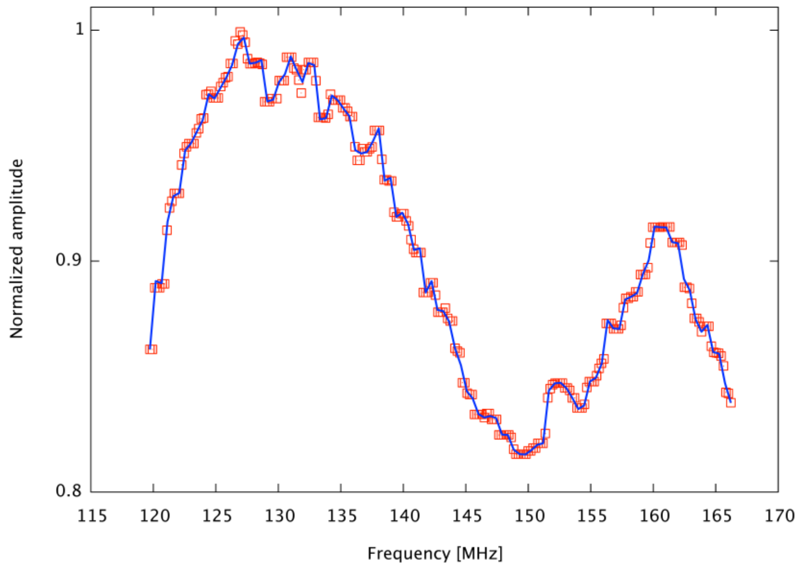
Frequency dependence of instrument response

Time-independent

Real valued Jones matrix:

$$\mathbf{J}_{i,\text{bandpass}} = \begin{pmatrix} A_{xx} & 0 \\ 0 & A_{yy} \end{pmatrix}$$





Primary beam

System response of instrument

Direction dependent

Can be directly calculated with `Everybeam`

Two elements

- Element beam (Full-Jones)
- Array factor (scalar term)

$$\mathbf{J}_{i,\mathrm{beam}} = a_{\mathrm{arr}} e^{i\phi_{\mathrm{arr}}} \begin{pmatrix} A_{xx} e^{i\phi_{xx}} & A_{xy} e^{i\phi_{xy}} \\ A_{yx} e^{i\phi_{yx}} & A_{yy} e^{i\phi_{yy}} \end{pmatrix}$$

Isolating effects - LoSoTo

Lofar **So**lution **To**olkit

Used for manipulating solutions

Solutions: for each time, frequency, polarization, find amplitude+phase part

Fit systematic effects in solutions

Fitting Polarization misalignment

Phase solutions (XX and YY)

Ionosphere and clock effects work on XX and YY identically

Polarization alignment causes a difference between XX and YY

LoSoTo `POLALIGN`:

- Takes long time average of XX-YY
- Reduces amount of free parameters

$$n_{\text{freqs}} \times n_{\text{time}} \times n_{\text{stations}} \times 2 \rightarrow n_{\text{stations}}$$

Fitting bandpass

Bandpass: amplitude effect (different for XX and YY)

LoSoTo `Smooth`

$$n_{\mathrm{freqs}} \times n_{\mathrm{time}} \times n_{\mathrm{stations}} \times 2 \rightarrow n_{\mathrm{stations}} \times n_{\mathrm{freqs}} \times 2$$

Tutorial

10 min. Calibrator scan of 3C295

Perform (part of) calibrator pipeline

Diagonal solve

$$\mathbf{J} = \begin{pmatrix} A_{xx}e^{i\phi_{xx}} & 0 \\ 0 & A_{yy}e^{i\phi_{yy}} \end{pmatrix}$$

Extract bandpass+polarization alignment

(Clock and ionosphere require separate treatment --> Talk by Maaike Mevius)

Software used

https://tikk3r.github.io/flocs/

https://dp3.readthedocs.io/en/stable/

https://github.com/revoltek/losoto/releases

Steps

Use FLOCS (https://tikk3r.github.io/flocs/)

Predict *(Converts skymodel to model visibilities)*

Solve *(Find solutions based on data and model visibilities)*

- *Note: Not completely accurate (no rotation solve)*

Extract polarization alignment with LoSoTo

Extract bandpass with LoSoTo