Introduction to Interferometry

Diffraction Limit and Aperture Synthesis.

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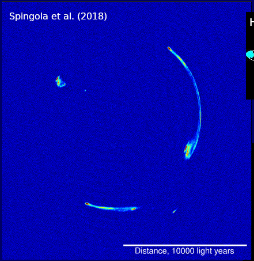
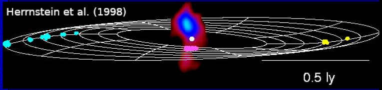
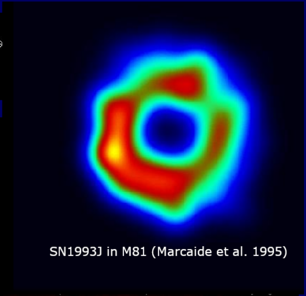
JIVE VLBI School 2025

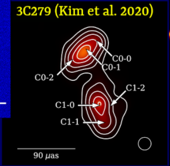
VNIVERSITAT DE VALÈNCIA

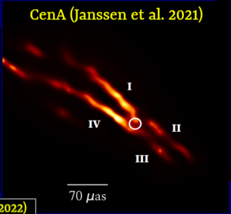
The Radio Universe at the Highest Resolutions

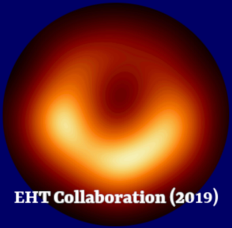


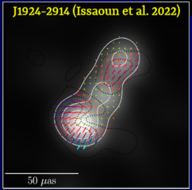


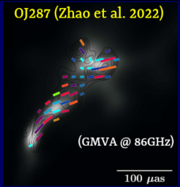




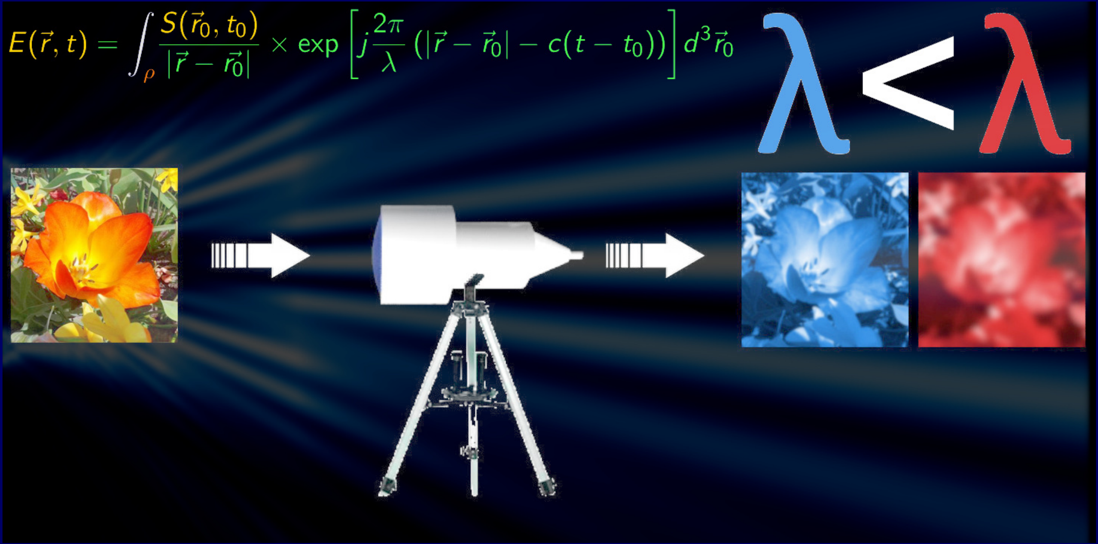








Light Diffraction



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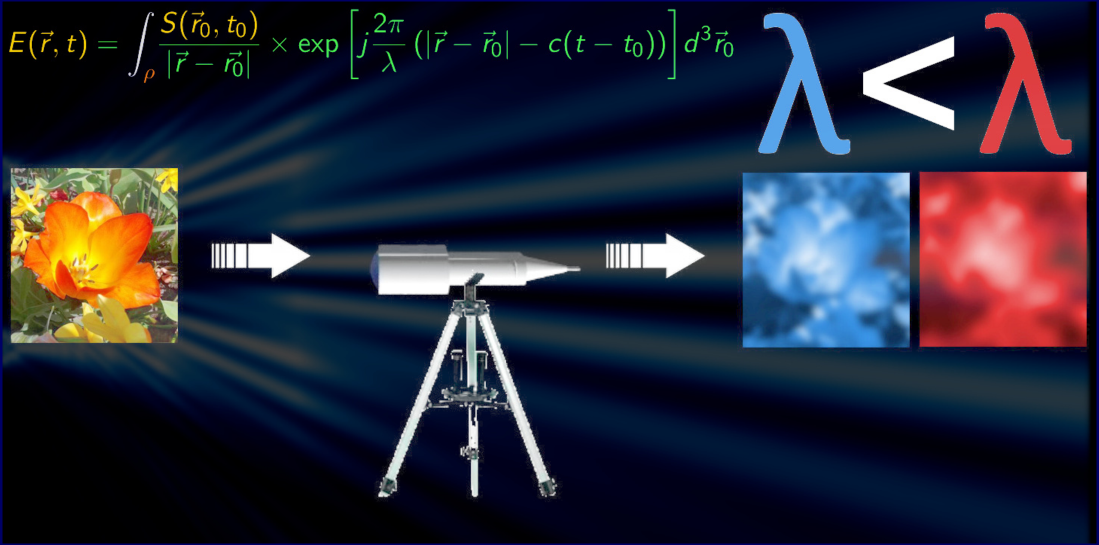


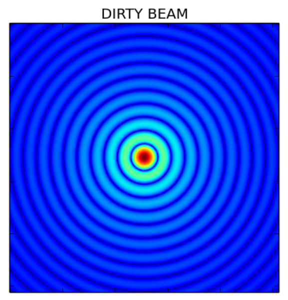
Resolution of an Optical Device

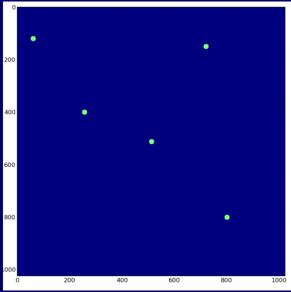
Any optical device is fundamentally limited by Diffraction

Due to Diffraction, the image of a point source is not a point image, but a blurry shape called Point Spread Function (PSF)

Resolution of an Optical Device

If we observe a source with a generic structure, the image obtained is the convolution of the source structure with the PSF





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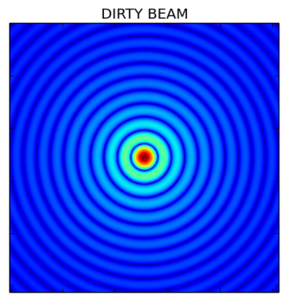
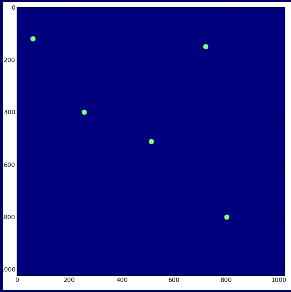
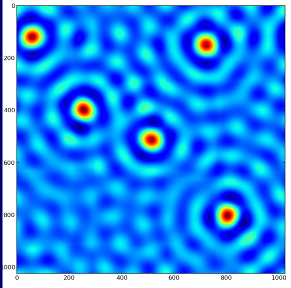






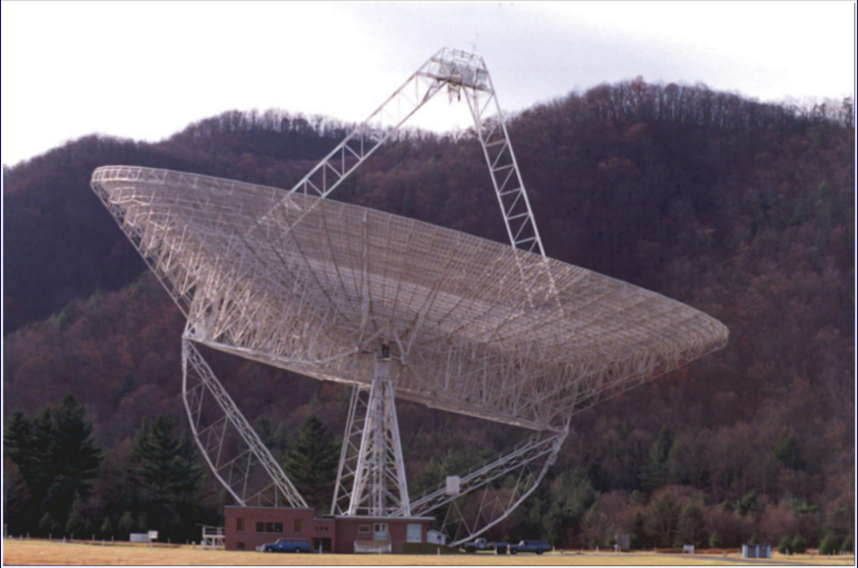
If the PSF is complicated (dirty), it may limit the maximum achievable contrast (dynamic range) in the image.

Aperture: a Fundamental Limitation

$$\Delta\theta \sim \lambda/D$$

The image resolution depends on the aperture
(measured in units of wavelength).

We Need Optical Aperture



We Need Optical Aperture

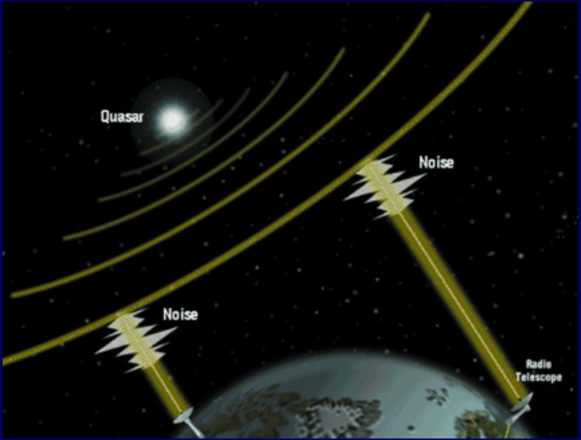
Diffraction Limit and Interferometry:

Hugging Your Enemy!

Spatial Coherence

Cred: Wayne Knox

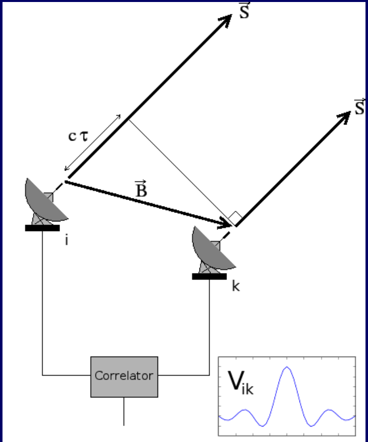
Extragalactic Ducks



We measure waves caused by *extragalactic ducks* in the *cosmic ocean*.

The Two-Element Interferometer

Visibility of a Monochromatic Source



E-field amplitude (point source): $E = A \exp\left[\frac{2\pi}{\lambda} j(\vec{S}\vec{x} - c\,t)\right]$

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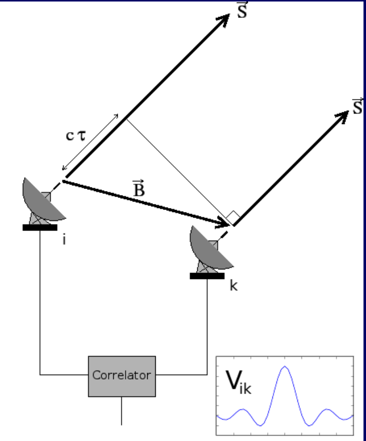

E-field amplitude (point source): $E = A \exp\left[\frac{2\pi}{\lambda} j(\vec{S}\vec{x} - c\,t)\right]$

The cross-spectrum (a.k.a. visibility) is the time cross-correlation between the electric fields received at the two antennas:

$V_{ik} = \langle E_i E_k^* \rangle = I \exp\left[\frac{2\pi}{\lambda} j \vec{S}\vec{B}_{ik}\right]$ with $\vec{B}_{ik} = \vec{x}_i - \vec{x}_k$ and $I = A^2$

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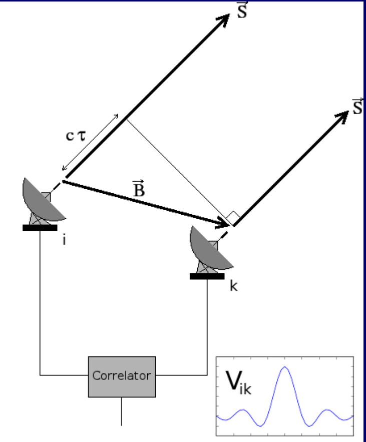

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If the source has several discrete components (of intensity I_m, located at $\vec{S}_m$):

$$V_{ik} = \sum_m \left(I_m \exp\left[\frac{2\pi}{\lambda} j \vec{S}_m \vec{B}_{ik}\right]\right)$$

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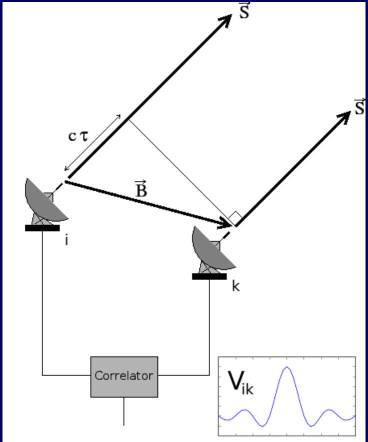

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For a continuous brightness distribution: $V_{ik} = \int_{\vec{S}} I(\vec{\Omega}) \exp\left[\frac{2\pi}{\lambda} j\vec{\Omega}\vec{B}_{ik}\right] d\vec{\Omega}$

Visibilities and Source Structure

Getting the Fourier Transform

This is what we have:

$$V_{ik} = \int_{\vec{S}} I(\vec{\Omega}) \exp\left[\frac{2\pi}{\lambda} j \vec{\Omega}\vec{B}_{ik}\right] d\vec{\Omega}$$

This is what we want:

$$V_{ik} = \int_{\vec{S}} I(\alpha', \delta') \exp\left[\frac{2\pi}{\lambda} j(u\alpha' + v\delta')\right] d\alpha' \, d\delta'$$

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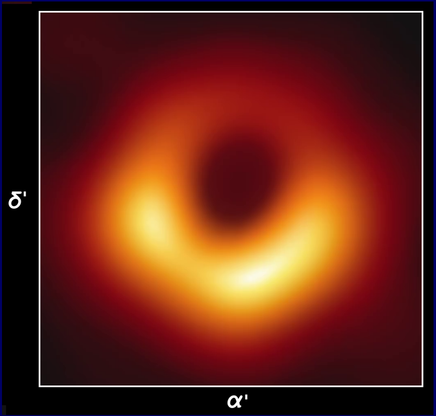

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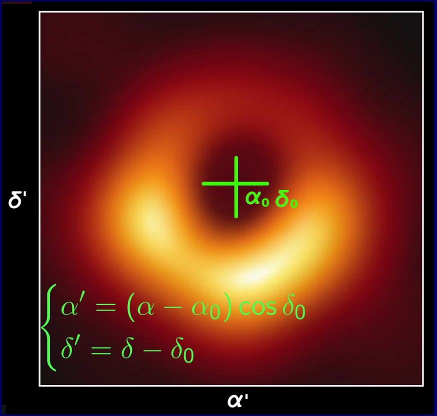

$$\begin{cases} \alpha' = (\alpha - \alpha_0)\cos\delta_0 \\ \delta' = \delta - \delta_0 \end{cases}$$

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- Write $\vec{\Omega}$ and $\vec{B}_{ik}$ in the right coordinate system: (U, V, W), where W points toward the source and (U, V) are parallel to the *director cosines*, (α', δ').

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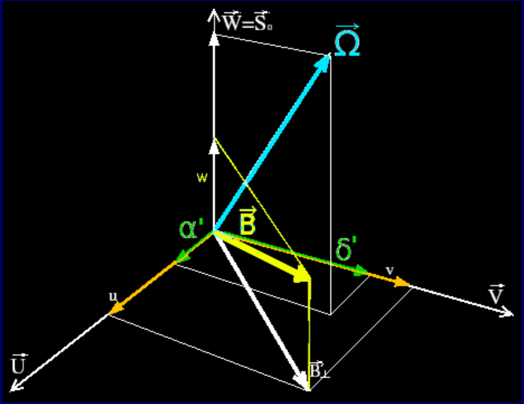

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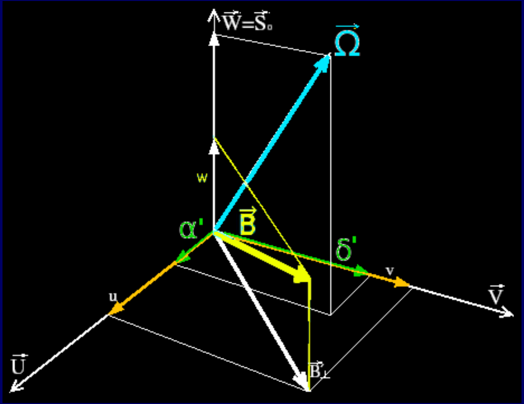

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- If $\sqrt{1 - (\alpha')^2 - (\delta')^2} \sim 1$, we can take this (constant) factor out of the integral!

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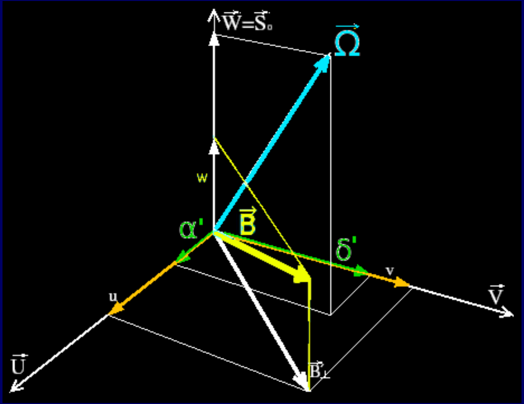
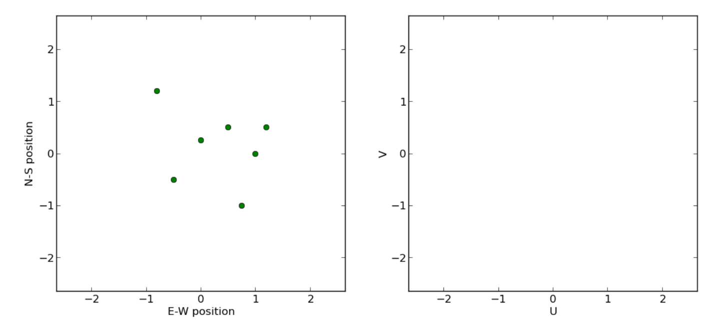


Multi-Element Interferometers

The UV Plane (Snapshot Observations)



Each pair of telescopes measures its own visibility. Each visibility is the Fourier transform of the source structure, measured at the points of the UV plane given by the projected baseline.

A multi-element interferometer is, thus, a set of *many two-element* interferometers!

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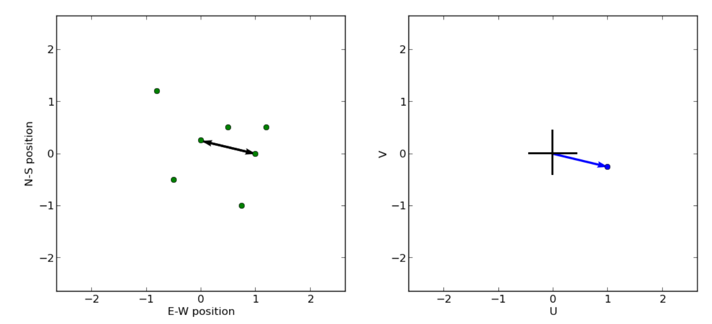

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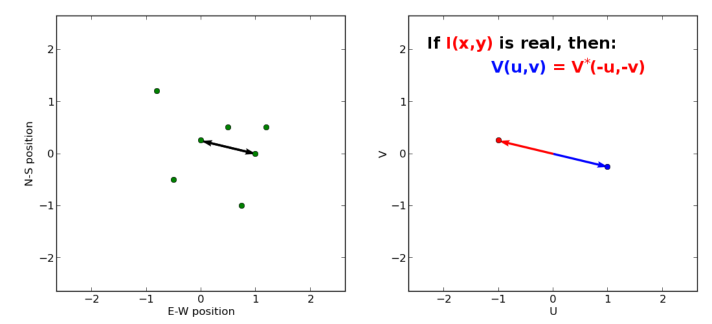

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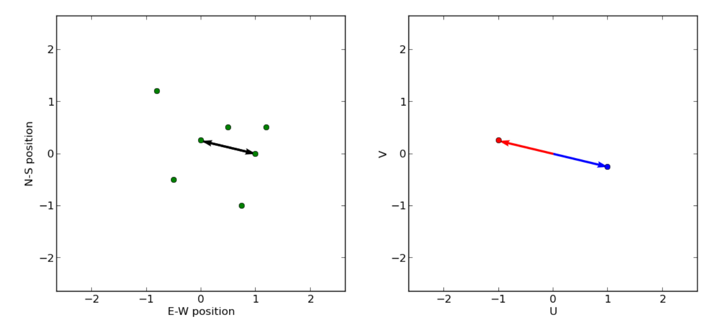

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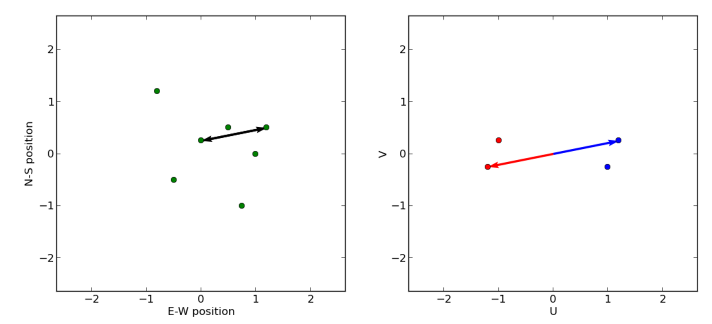

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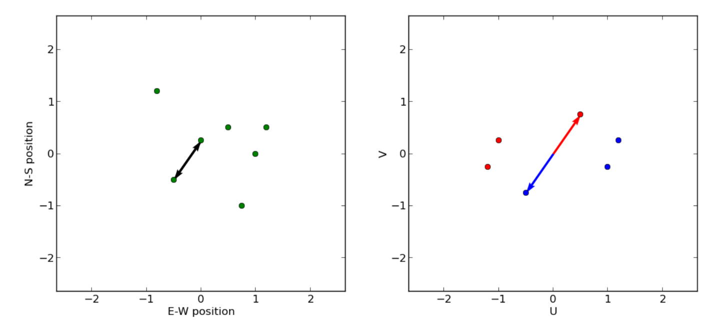

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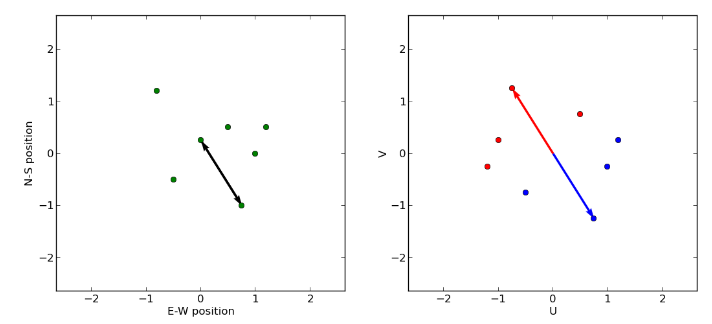

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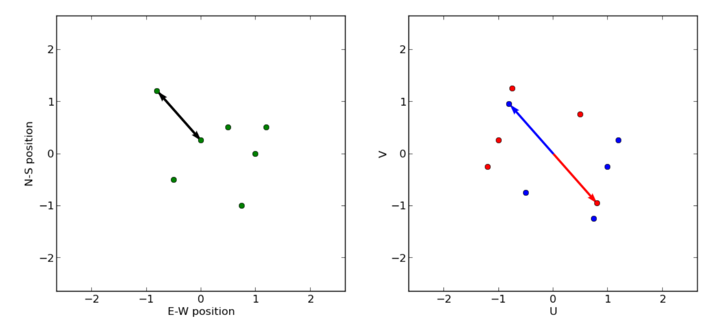

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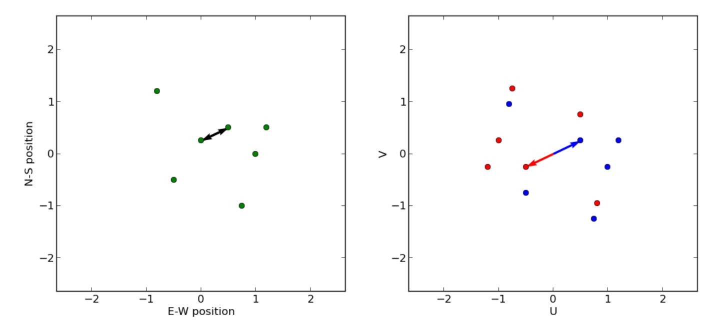

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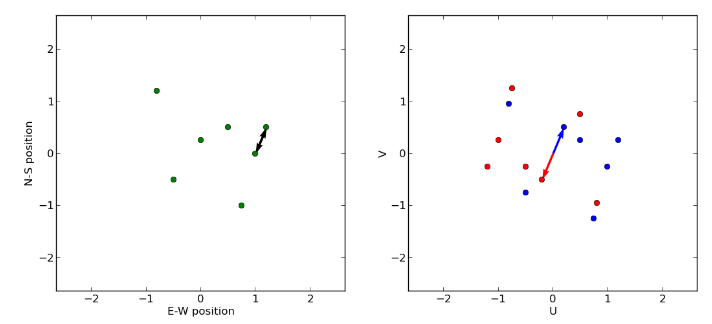

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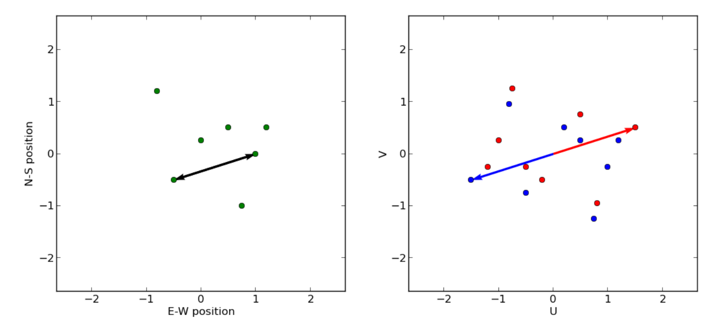

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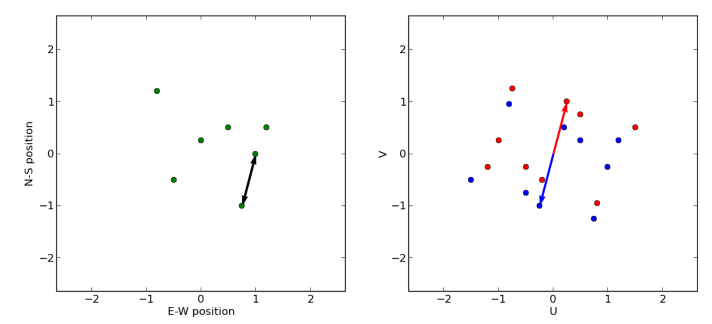

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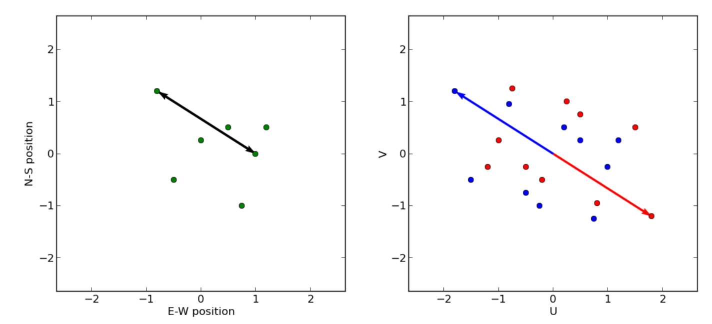

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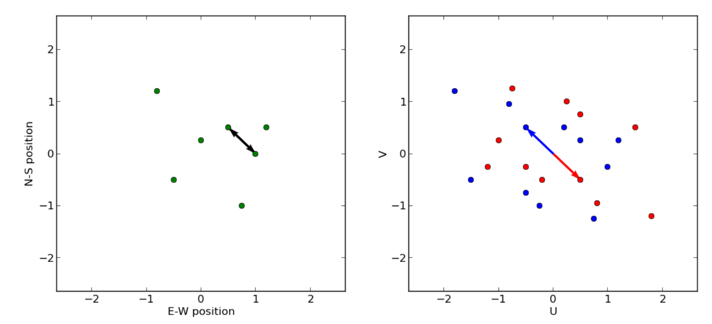

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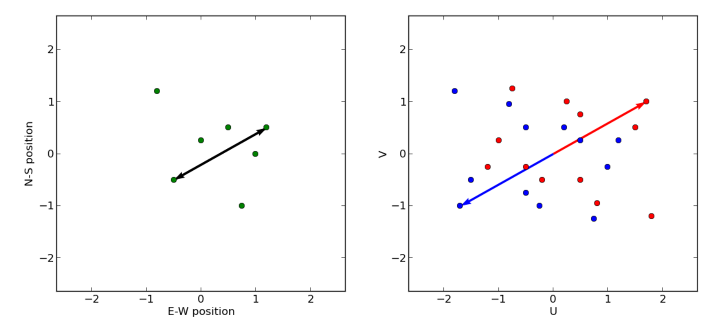

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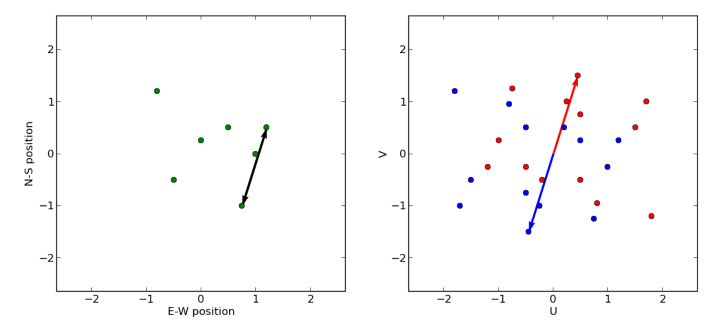

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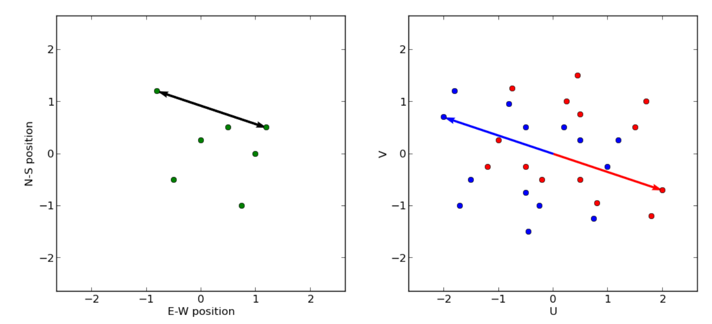

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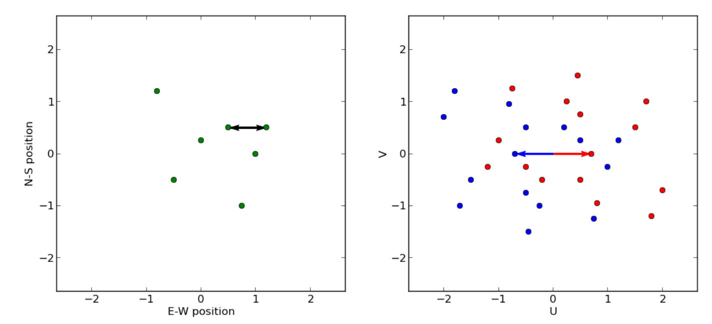

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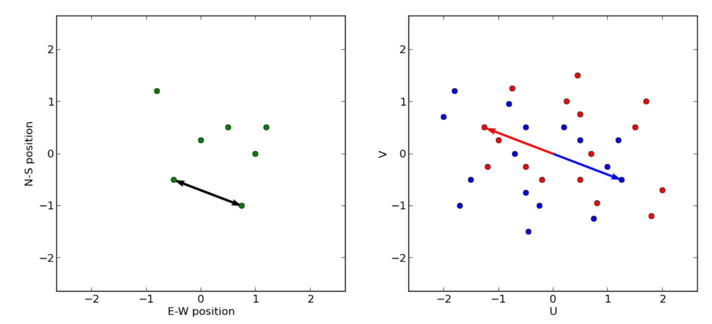

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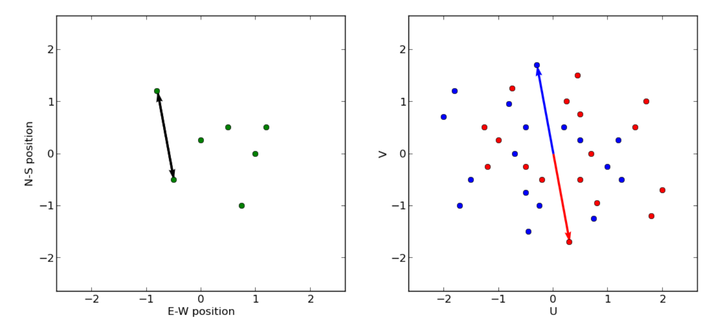

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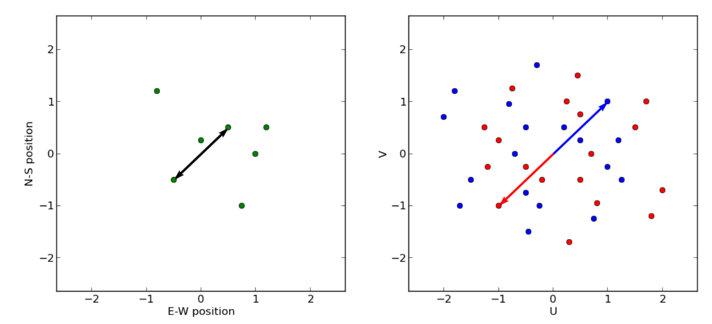

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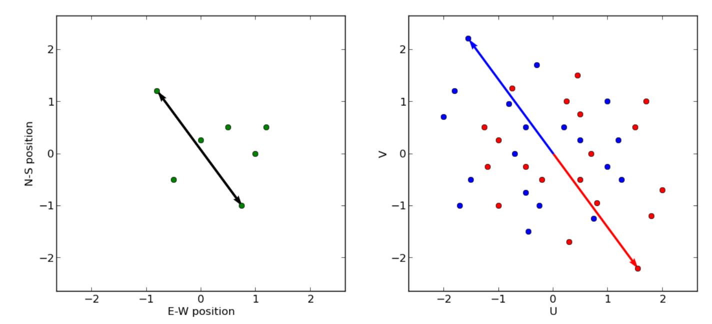

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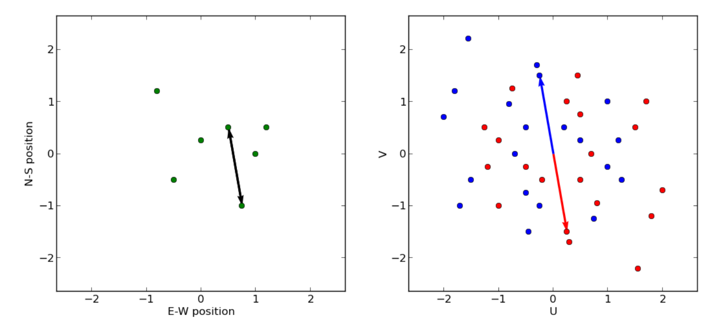

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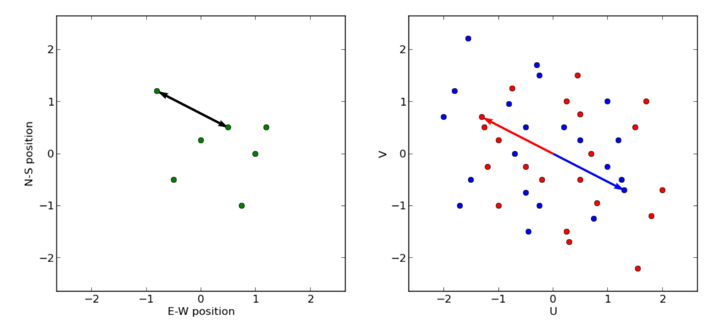

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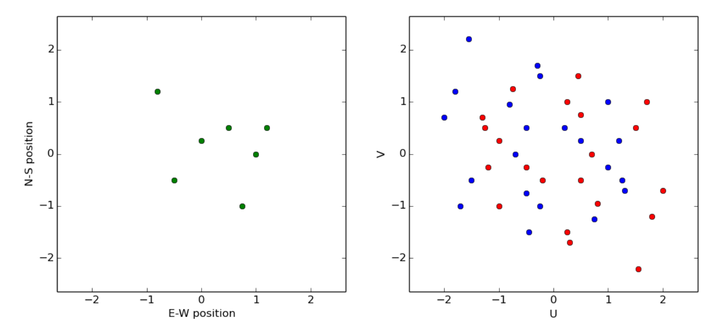

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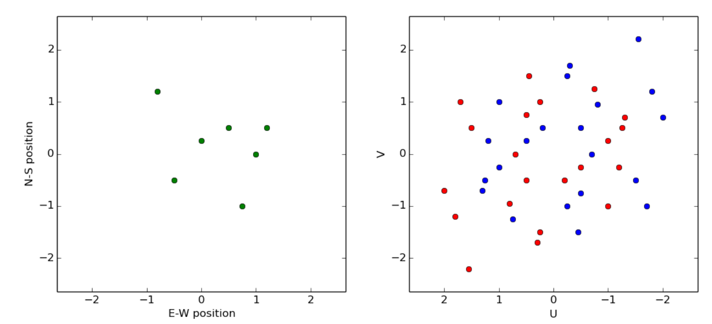

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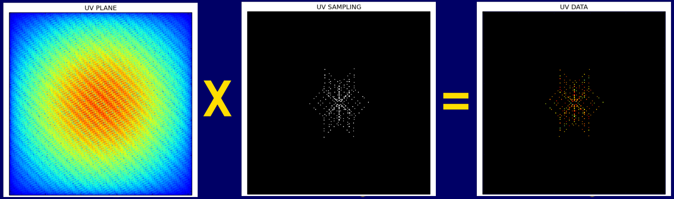
BEWARE! The U axis must point to East (like the Right Ascension!)

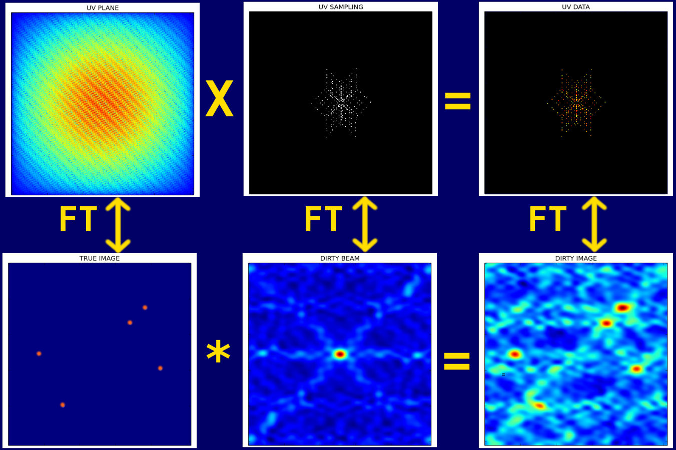
Image Reconstruction

Aperture Synthesis: the PSF



Our Measurements are equal to the Visibility Function multiplied by the UV coverage (i.e., the Sampling Function).

Aperture Synthesis: the PSF



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Our Image is equal to the True Image convolved with the Point Spread Function.

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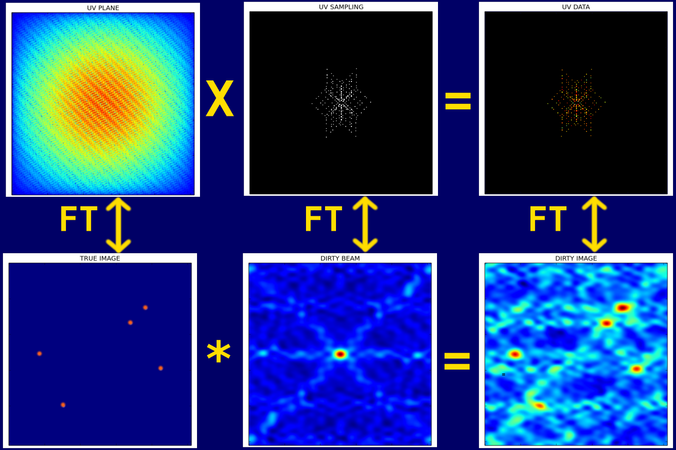

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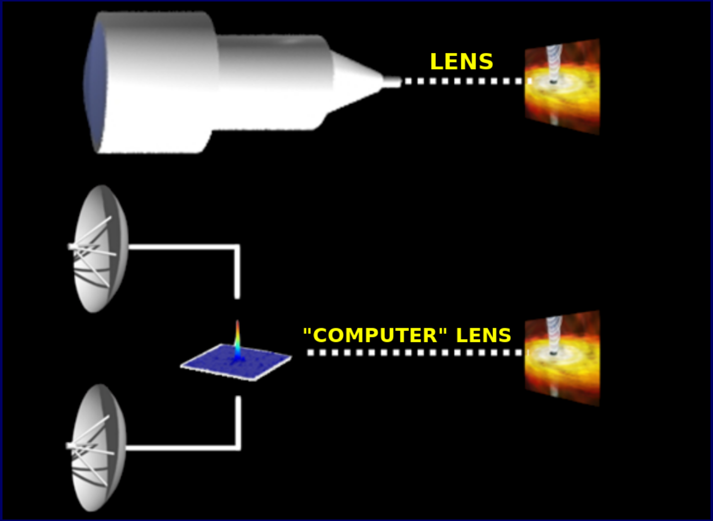
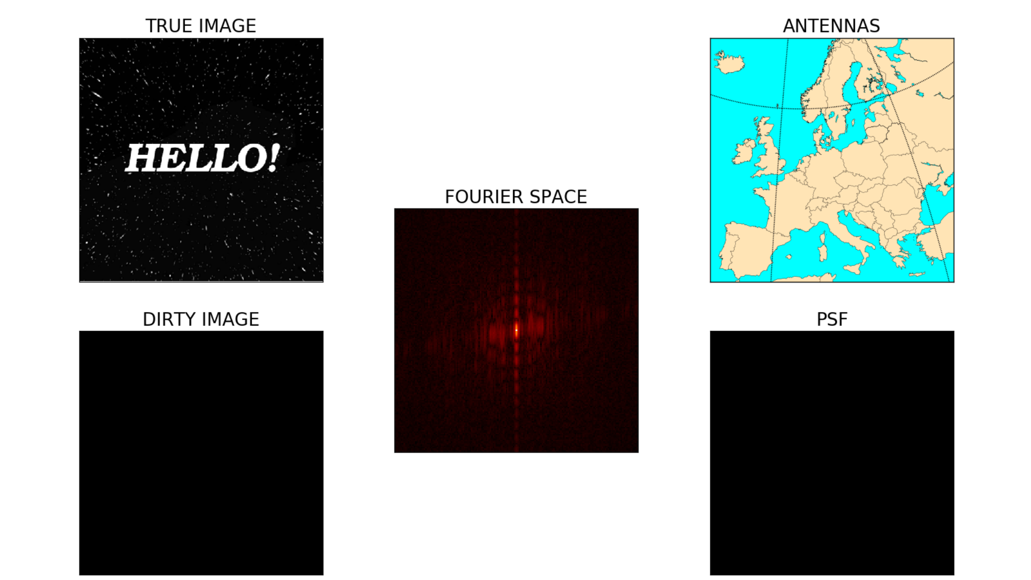
Therefore, the PSF is the Fourier Transform of the UV Coverage!

An Interferometer Seen as a Telescope



Earth Rotation Aperture Synthesis



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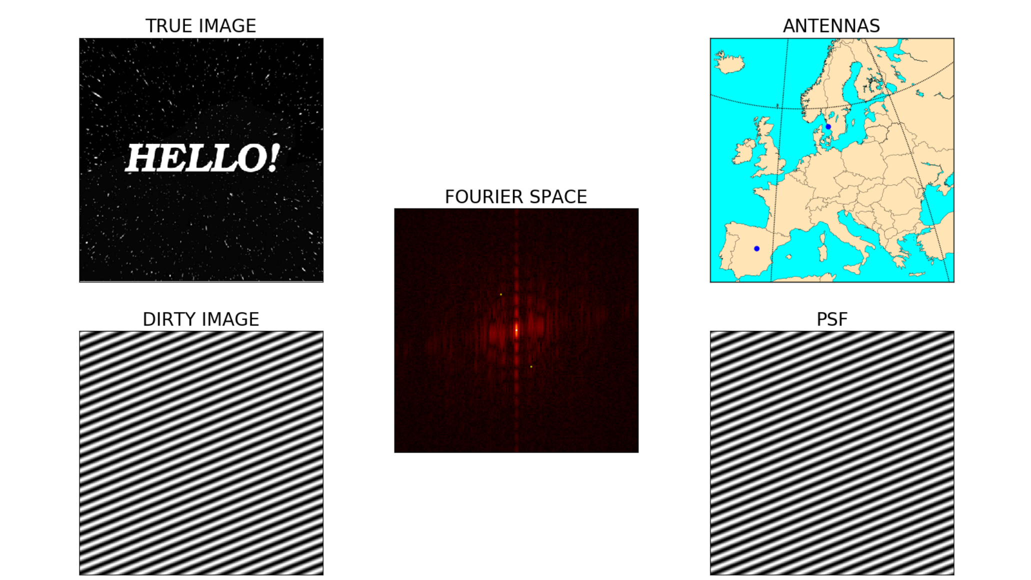

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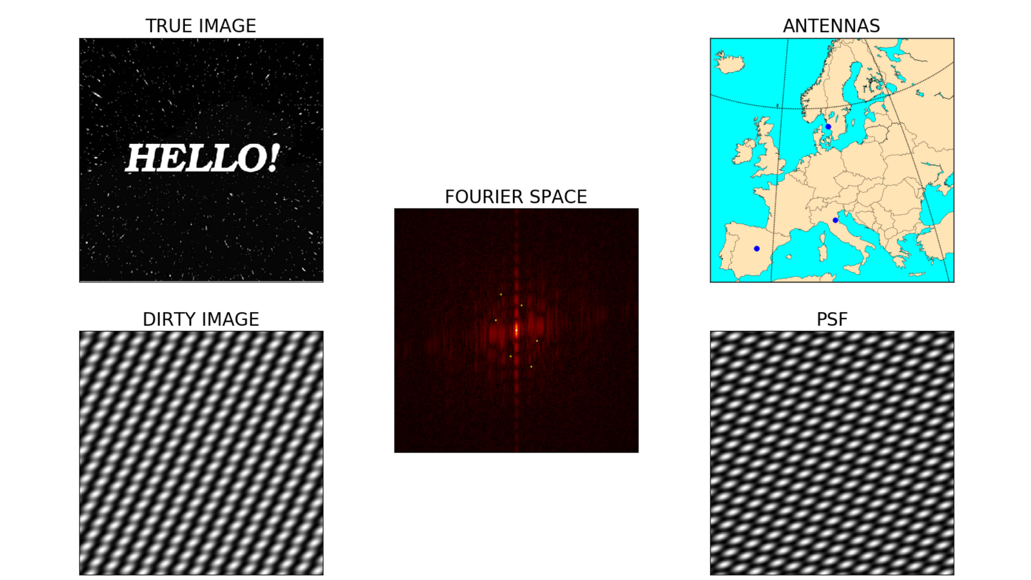

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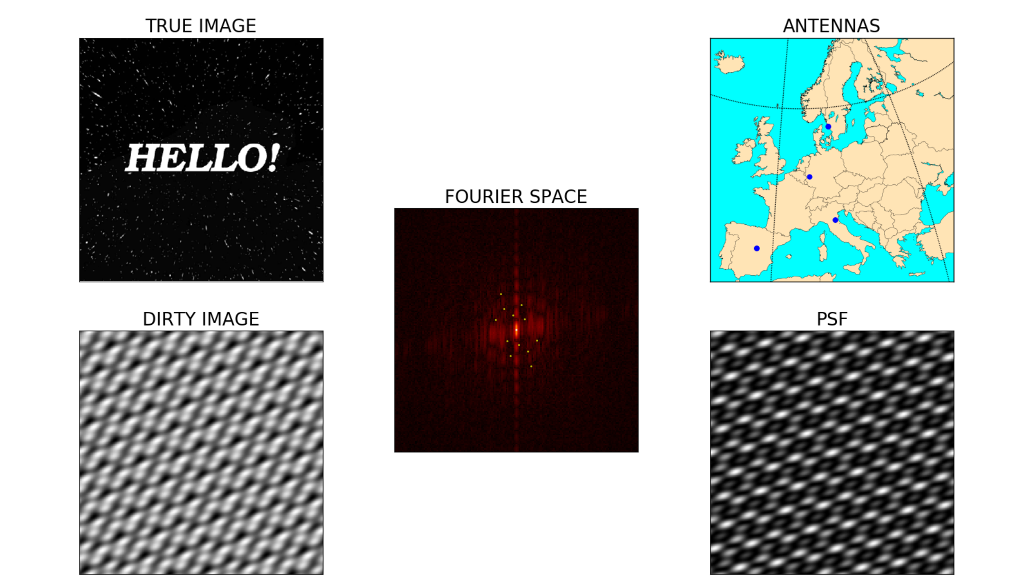

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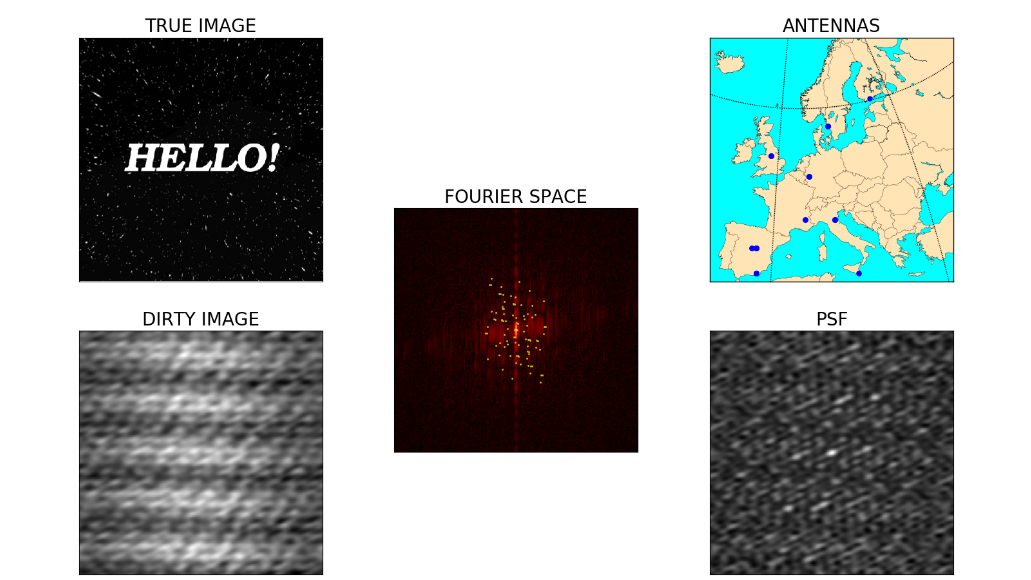

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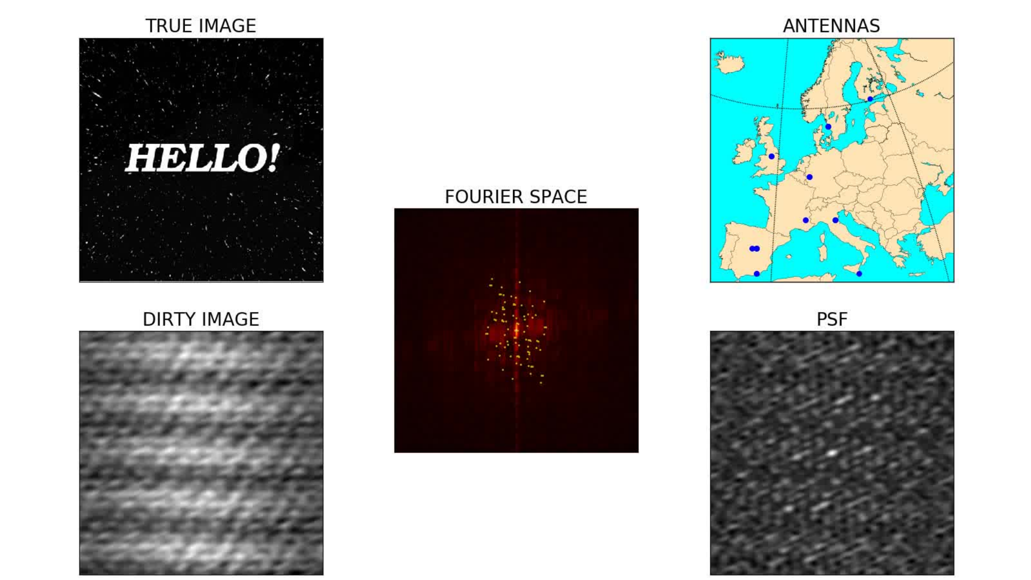

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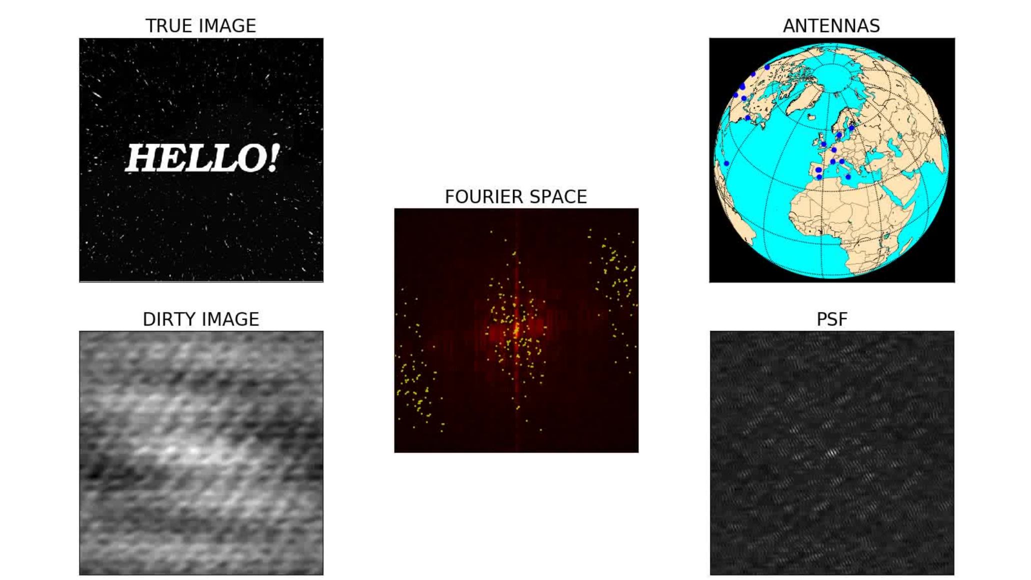

Summary

- Diffraction Limit is present in all optical devices: $\theta \propto \lambda/D$.
- Diffraction is also related to the spatial coherence of light.
- An interferometer is a device that measures the spatial coherence: visibilities.
- The visibilities are related to the source brightness distribution via a Fourier transform (with some assumptions, e.g. small-field approximation).
- The instrumental response to a point source (PSF) is related to the distribution of (projected) baselines during the observations. Earth rotation improves the PSF.

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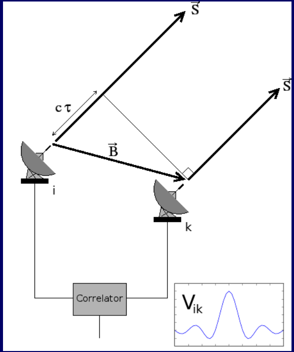
- The instrumental response to a point source (PSF) is related to the distribution of (projected) baselines during the observations. Earth rotation improves the PSF.
- Things that we still need to learn (incomplete list!):
  - How to deal with the effects of noise, atmosphere, Earth-model inaccuracies, antenna receivers, etc. on the visibilities.
  - Techniques to "remove" the effects of the PSF from the reconstructed images.
  - How to deal with spectral lines, polarization, ...

The two-element interferometer



BASIC OBSERVABLE: **CROSS-SPECTRUM** (a.k.a. **COMPLEX VISIBILITY**).

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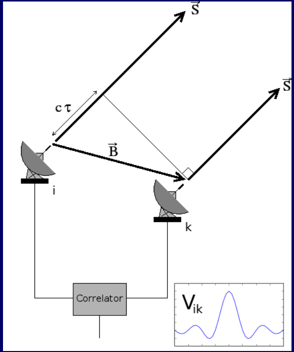

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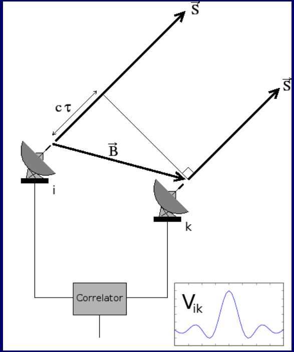

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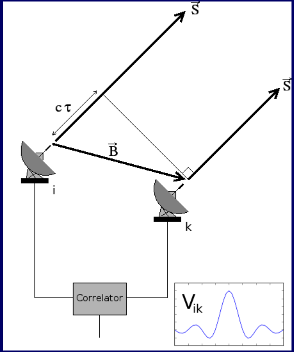

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FX ALGORITHM:

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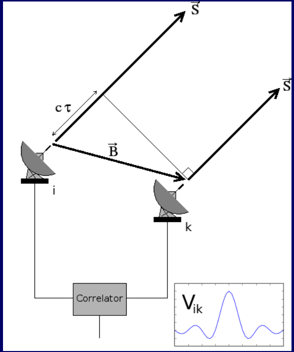

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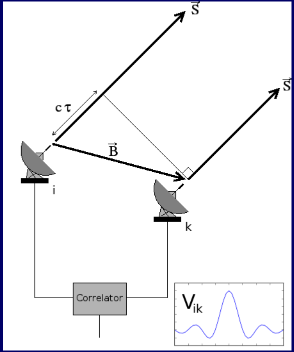

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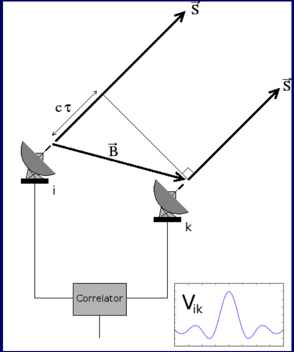

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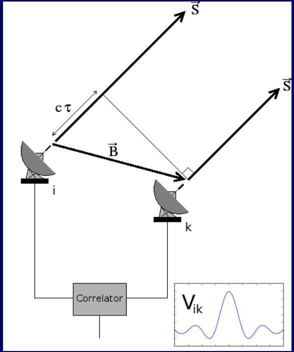

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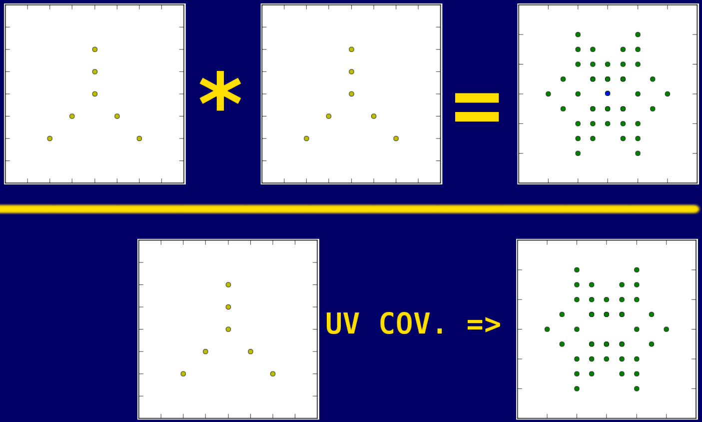
Notice the similarity with the power spectrum used in single dish: $P_i(\nu) = S_i(\nu) \times S_i^*(\nu) = |S_i(\nu)|^2$

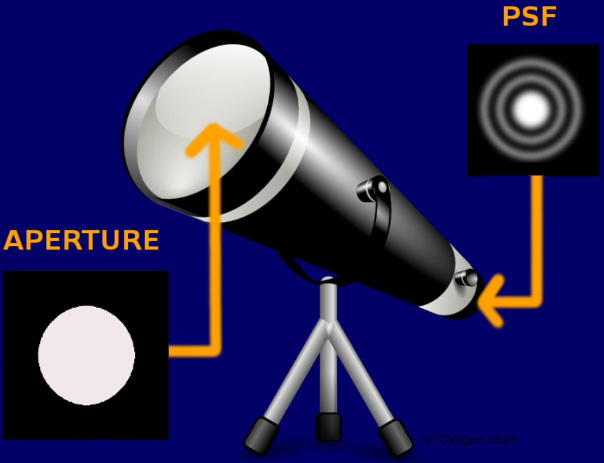
The Snapshot UV coverage

The UV coverage in a snapshot is *almost* equal to the autocorrelation of the antenna distribution, as it is seen from the source.



Aperture of an Interferometer's Snapshot

For a snapshot, the interferometer is *almost* like masking the aperture of a huge telescope!



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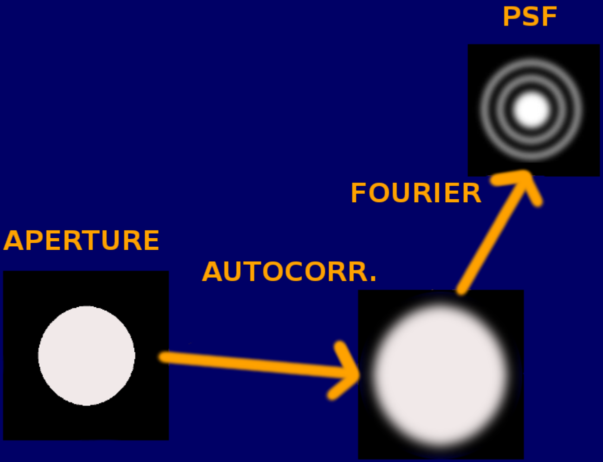

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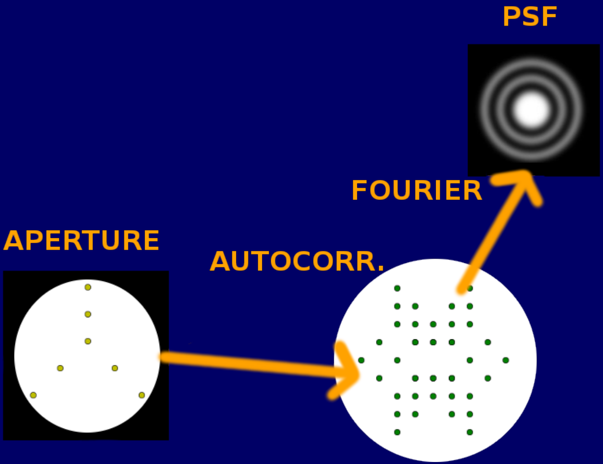
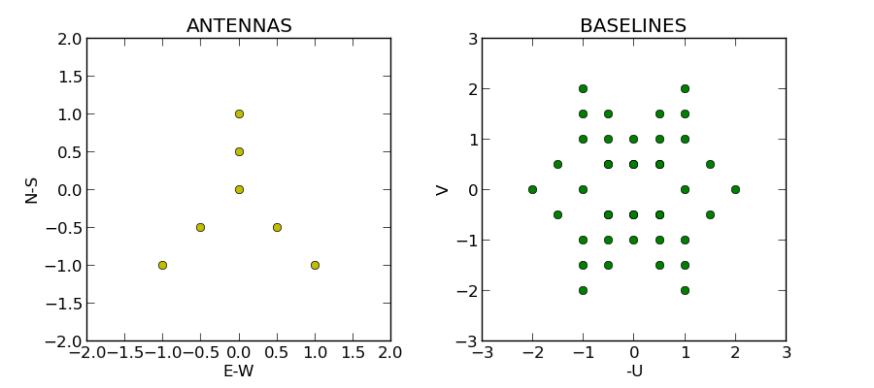


Examples of Snapshot UV Coverages



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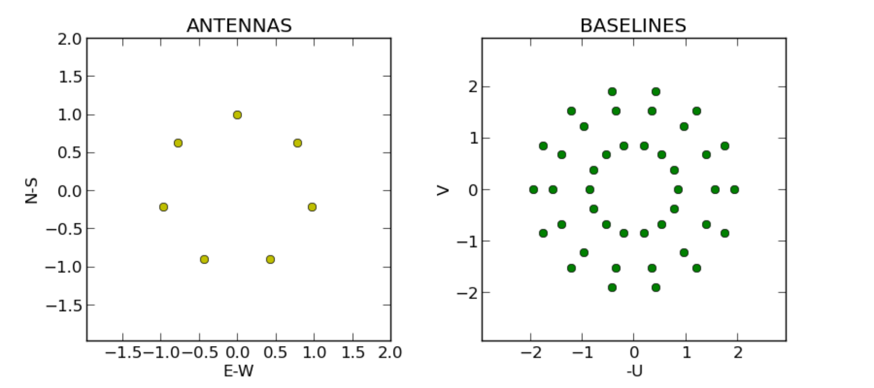

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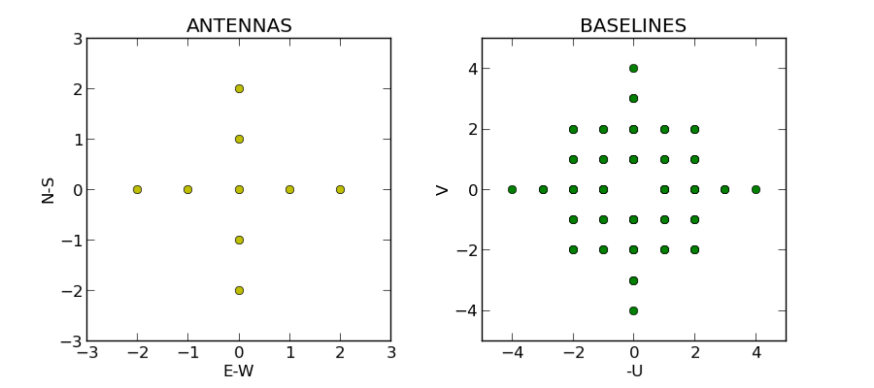

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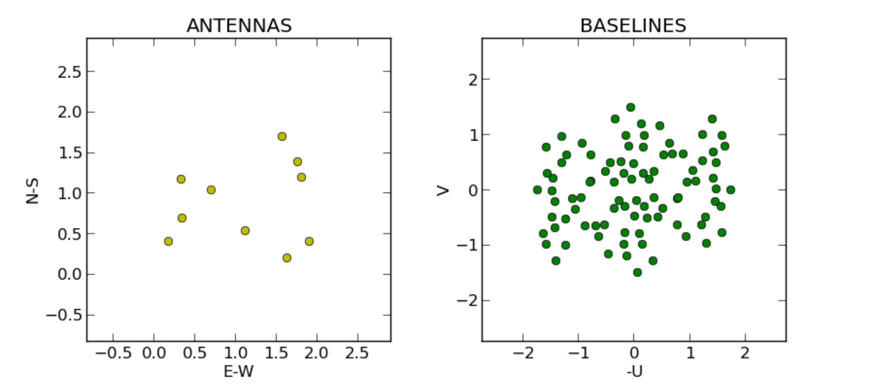

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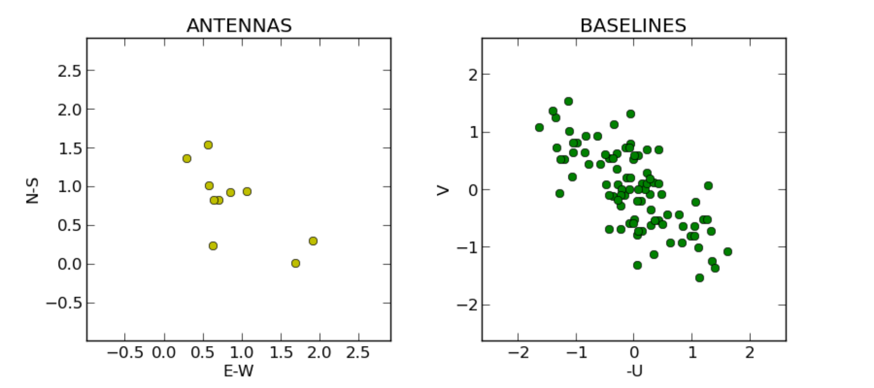

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