# **Basics on Polarization Observations**

Introduction to Interferometric Polarimetry

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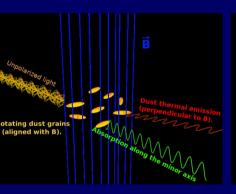
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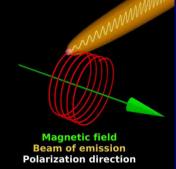
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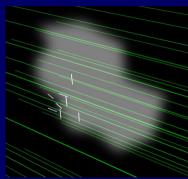


# Light polarization in the Universe.



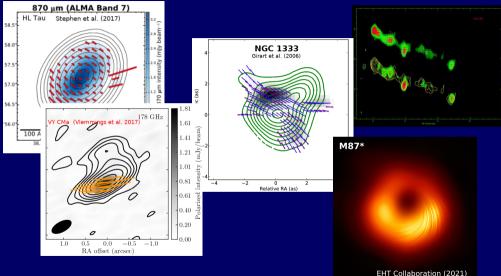






# Polarized light carries a lot of information!





3 / 14

### **Detecting source polarization**

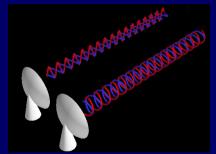


• How do we detect the orientation of the electric field with an antenna?

# **Detecting source polarization**



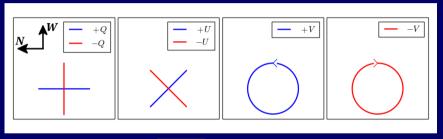
- How do we detect the orientation of the electric field with an antenna?
- Polarizing receivers (polarizers). The signal is split coherently into two orthogonal polarization states.
  - Linear polarizers (horizontal / vertical linear polarization).
  - Circular polarizers (left / right circular polarization).



# The Stokes parameters



• The Stokes parameters: I, Q, U, and V



- Linear polarization:  $I_p = \frac{\sqrt{Q^2 + U^2}}{I}$ ,  $\theta = \frac{1}{2} \arctan\left(\frac{U}{Q}\right)$
- Unpolarized intensity:  $I_u = \sqrt{I^2 Q^2 U^2 V^2}$

# The Measurement Equation





• We measure the signal cross-correlations between radio telescopes, a and b.





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- We compute all combinations of polarization cross-correlations (a.k.a. visibilities):
  - ▶ The so-called "parallel hands":  $V_{RR}^{ab} = \langle R^a \times (R^b)^* \rangle$  and  $V_{LL}^{ab} = \langle L^a \times (L^b)^* \rangle$ .
  - ▶ The so-called "cross hands":  $V_{RL}^{ab} = \langle R^a \times (L^b)^* \rangle$  and  $V_{LR}^{ab} = \langle L^a \times (R^b)^* \rangle$





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- These cross-correlations can be related to the Stokes parameters of the observed source.

# The RIME (e.g., Smirnov 2011)





This is what we measure:

 $R^a$ ,  $L^a$ ,  $R^b$ ,  $L^b$ 



#### This is what we want:

 $\mathcal{I}(\alpha, \delta)$ ,  $\mathcal{Q}(\alpha, \delta)$ ,  $\mathcal{U}(\alpha, \delta)$ ,  $\mathcal{V}(\alpha, \delta)$ 

# The RIME (e.g., Smirnov 2011)





#### This is what we measure:

 $R^a$ ,  $L^a$ ,  $R^b$ ,  $L^b$ 

#### **Visibility Matrix:**

$$V_{\odot}^{ab} = \left[egin{array}{cc} V_{RR}^{ab} & V_{RL}^{ab} \ V_{LR}^{ab} & V_{LL}^{ab} \end{array}
ight]$$



#### This is what we want:

 $\mathcal{I}(\alpha,\delta)$ ,  $\mathcal{Q}(\alpha,\delta)$ ,  $\mathcal{U}(\alpha,\delta)$ ,  $\mathcal{V}(\alpha,\delta)$ 

#### **Brightness Matrix:**

$$S_{\odot} = \left[ egin{array}{ccc} \mathcal{I} + \mathcal{V} & \mathcal{Q} + j\mathcal{U} \ \mathcal{Q} - j\mathcal{U} & \mathcal{I} - \mathcal{V} \end{array} 
ight]$$

# The RIME (e.g., Smirnov 2011)





#### This is what we measure:

$$R^a$$
,  $L^a$ ,  $R^b$ ,  $L^b$ 

#### **Visibility Matrix:**

$$V_{\odot}^{ab} = \left[egin{array}{cc} V_{RR}^{ab} & V_{RL}^{ab} \ V_{IR}^{ab} & V_{II}^{ab} \end{array}
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#### This is what we want:

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#### Radio Interferometer Measurement Equation (VLBI case):

$$V_{\odot}^{ab} = J_a \left( \int S_{\odot}(\alpha, \delta) \exp \left[ 2\pi j \frac{u\alpha + v\delta}{\lambda} \right] d\alpha d\delta \right) J_b^H$$
 where  $J_a$  and  $J_b$  are gain matrices.

7 / 14

# The MEq. A Full-Stokes Formalism



For a source with a generic structure, the visibility matrix for antennas a and b (with no direction-dependent calibration) is:

$$V^{ab} = J_a \left[ \int_{\alpha,\delta} S \, e^{-rac{2\pi j}{\lambda} (u \, \alpha + v \, \delta)} \, rac{dlpha \, d\delta}{z} 
ight] (J_b)^H,$$

Let us remember the classical equation (where  $V^{ab}$  was a complex scalar, not a matrix):

$$V^{ab} = G_a G_b^* \int_{\alpha,\delta} I(\alpha,\delta) e^{-\frac{2\pi j}{\lambda}(u\alpha+v\delta)} \frac{d\alpha d\delta}{z}$$

# Jones calibration matrices. Examples



• Gain, 
$$G = \begin{pmatrix} A_r(t) e^{j\phi_r(t)} & 0 \\ 0 & A_l(t) e^{j\phi_l(t)} \end{pmatrix}$$

• Delay, 
$$K=egin{pmatrix} e^{j au_r(
u-
u_0)} & 0 \ 0 & e^{j au_l(
u-
u_0)} \end{pmatrix}$$

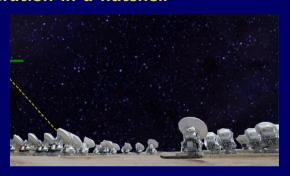
• Bandpass, 
$$B = \begin{pmatrix} A_r(\nu) e^{j\phi_r(\nu)} & 0 \\ 0 & A_l(\nu) e^{j\phi_l(\nu)} \end{pmatrix}$$

• Polarization Leakage (a.k.a. "Dterms"), 
$$D = \begin{pmatrix} 1 & D_r(\nu) \\ D_r(\nu) & 1 \end{pmatrix}$$

Most Jones matrices are diagonal (good!). They are also multiplicative, e.g.:  $J = G \times B \times K$ , but care must be taken, since (non-diagonal) matrices generally do not commute.

#### Polarization calibration in a nutshell





- The axes of the antenna mounts are "tied" to the Earth (green).
   So are their polarizers.
- The source orientation is tied to the sky (yellow).
- Since the signal in a polarizer depends on its orientation w.r.t. the source, the Earth rotation allows us to decouple instrumental effects from the source polarization.

10 / 14

#### Polarization calibration



- Parallactic angle (rotation of antenna axes w.r.t. the sky. It is *deterministic*, so it should be done *a-priori*).
- Polarization leakage (the most tricky instrumental polarization quantity; it is usually done at the very end, after a final total-intensity image is obtained).
- Cross-Delay/phase (used to account for the optical-path difference between polarizations; should be done in a preliminary fringe-fitting).
- Amplitude ratio (hopefully calibrated with gains ant Tsys; should be done a-priori).

### **Calibration strategy**



The right order for matrix product is:  $J = (G_a K_c) \times D \times P \times (G K)$ 

i.e.: 
$$V^{cal} = (G K)^{-1} \times P^{-1} \times D^{-1} \times (G_a K_c)^{-1} \times V^{obs}$$

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- STEP 0: Apply parallactic-angle correction and cross-polarization amplitude ratio.
- STEP 1: Calibrate the cross-delay(phase) using a strongly polarized source.
- STEP 2: Calibrate the leakage using an unpolarized source.
  - ▶ If all calibrators are polarized, solve for leakage and source polarization simultaneously.
  - Need good parallactic-angle coverage.
- STEP 3: Image each Stokes parameter separately. Combine images:  $(Q, U) \rightarrow (I_p, \theta)$ .

# Calibration strategy



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- STEP 0: Apply par
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 $P \times (G K)$ 

 $V^{obs}$ 

amplitude ratio.

d source.

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EHT Collaboration (2021)

12 / 14

# **VLBI Polarimetry Software**

### **VLBI Polarimetry Software**



Nearly all the polarization calibrator sources have resolved structures in VLBI The problem of using spatially-resolved polarization calibrators is that we need to estimate the D and (complex) S matrices at the same time.

#### Inverse Modelling.

- ▶ LPCAL (Leppänen et al. 1995) for AIPS. Pretty old, but well established and tested.
- ▶ GPCAL (Park et al. 2020) for AIPS. Overcomes some LPCAL limitations.
- ▶ PolCal (Moellenbrock) for CASA. Some limitations critical for VLBI.
- ▶ PolSolve (Martí-Vidal et al. 2020) for CASA. It overcomes almost all the LPCAL & PolCal limitations.

#### Forward Modelling.

**EHTim** (A. Chael et al. 2018, 2020)

#### MCMC.

▶ DMC (D. Pesce 2020) and THEMIS (Broderick et al. 2020)

#### **SUMMARY**



- We have reviewed basic concepts of polarization.
  - Modes of polarization.
  - Stokes parameters.
- We have discussed about the different kinds of polarizers in radioastronomical receivers.
  - Linear polarizers (X-Y).
  - Circular polarizers (R-L).
- We have studied how to deal with polarization in interferometric observations.
  - The Measurement Equation.
  - The matrices for polarization calibration.
  - Overview of calibration procedure.