









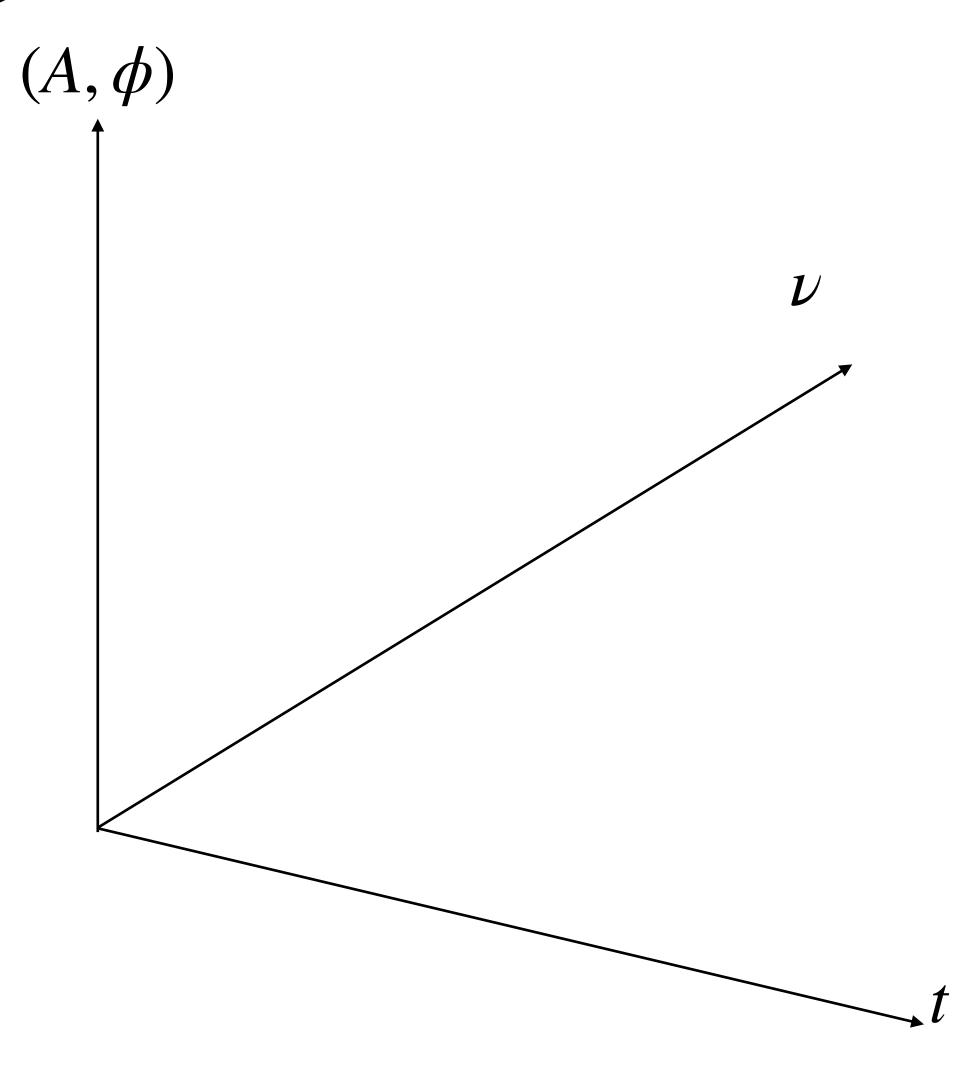


Outline

- 1. Interferometric data structure
- 2. Error sources
- 3. The calibration strategy
- 4. A priori calibration
- 5. Flux scaling
- 6. Fringe fitting
- 7. Bandpass corrections
- 8. Phase referencing

Interferometric data structure

- Our interferometer measures the FT of the sky (van Cittert-Zernike theorem) so each baseline measures an amplitude and phase.
- It is best to visualise the interferometric data in 3-D where your axes are:
 - Amplitude & phase ('complex gains')
 - Frequency
 - Time
- That's a lot of data



Interferometric data structure

 Our radio interferometer equation took no account of the frequency dependence (or time) of the incoming radiation:

$$V_{pq} = \iint_{lm} \mathbf{B} \exp \left\{ -2\pi i \left(u_{pq} l + v_{pq} m + w_{pq} (n-1) \right) \right\} dl dm$$

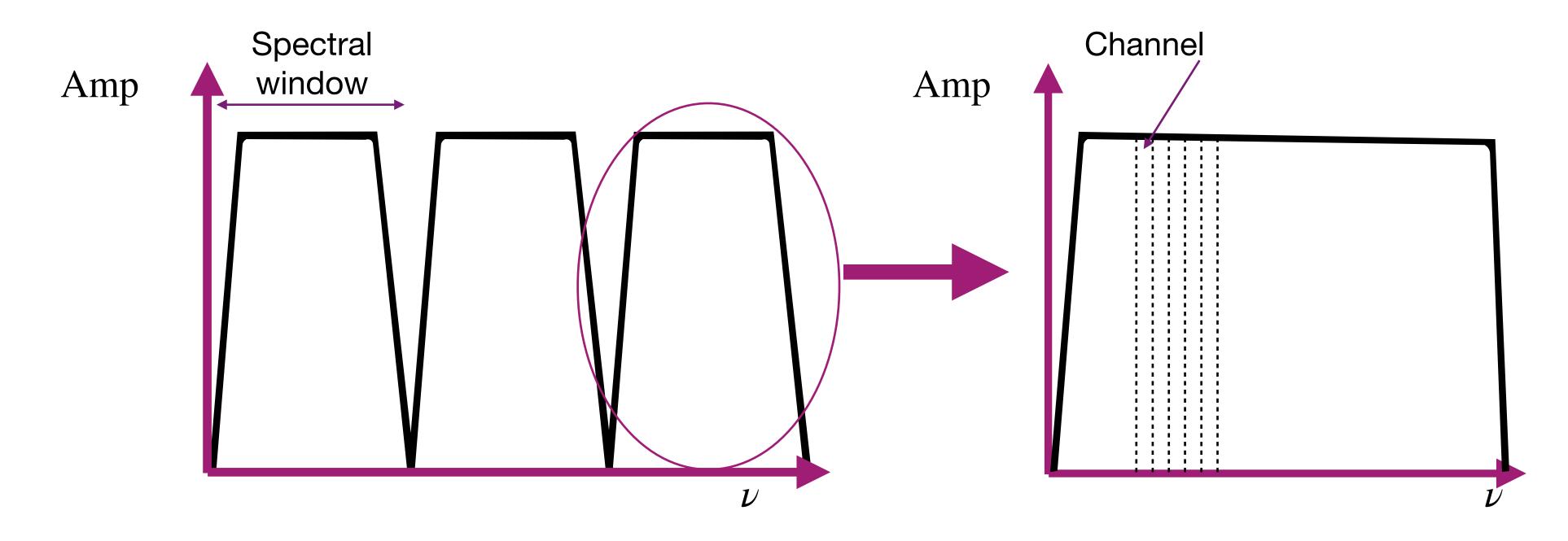
• In fact, the geometric delay will be different at different frequencies (and times) i.e.

$$2\pi\lambda^{-1}\left(u_{pq}l + v_{pq}m + w_{pq}(n-1)\right)$$

- To recover the sky brightness distribution we need to take this into account. Best way is to assume monochromatic radiation in small sub-bands (and sub-times) and invert each one.
- As a result the response of an interferometer in frequency and time space is split.

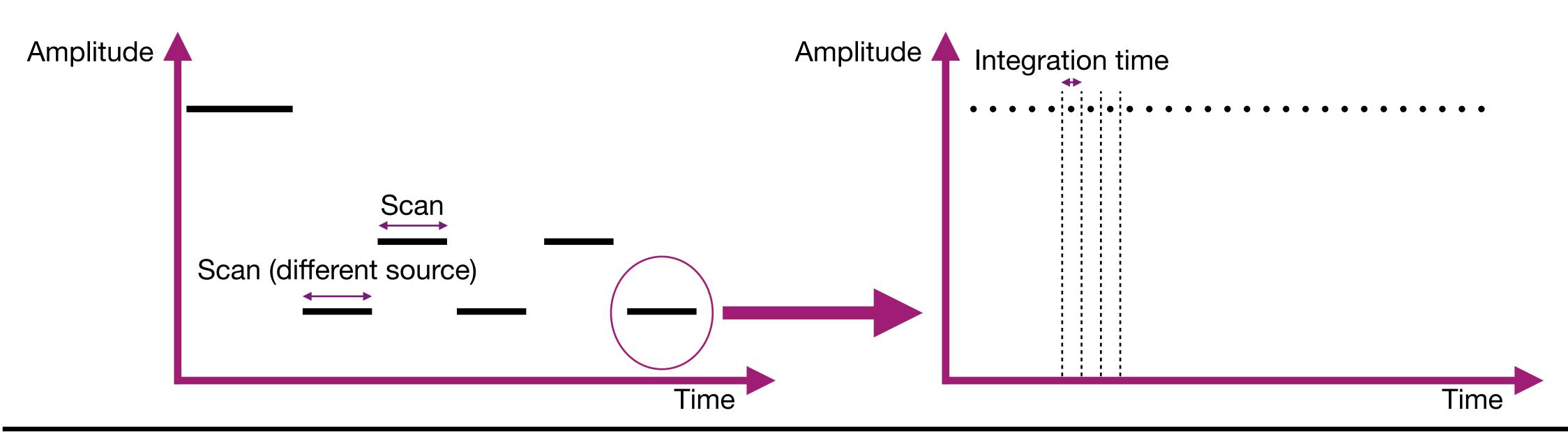
Interferometric data - frequency structure

- The frequency structure is split into two sub-band categories:
 - Semi-wide **spectral windows** these are governed by the receiver of the antennas (and/or correlator FTs and filters), and typically cover regions where the receiver is most sensitive.
 - Each spectral window is then split further into **channels**. This are small regions of frequency space where we can assume monochromatic radiation (we will learn about when this breaks down when we talk about bandwidth smearing)



Interferometric data - time structure

- In time space it is much simpler, the data is chunked into time integrations so that the geometric delay doesn't change too much across the averaging time.
- These integrations are grouped into scans, which describes which source the telescope is pointing at.
- For both frequency and time, there is a trade off between data rates & accuracy of the FT inversion (i.e. recovering the sky brightness)

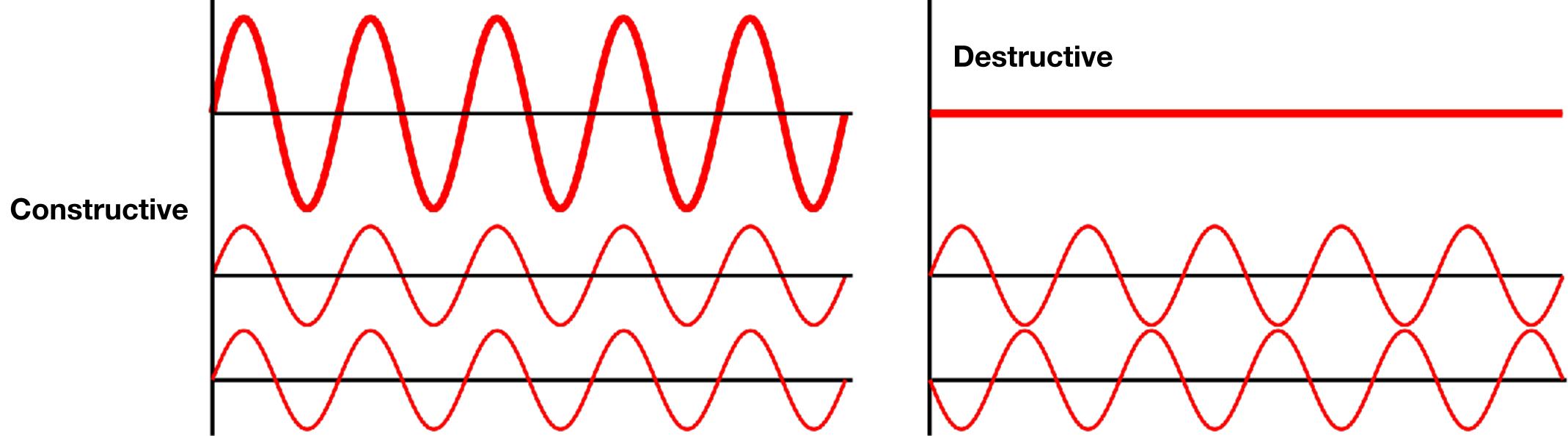


Corruption of interferometric data

- An interferometer interferes the signal coming into various antennas at once. However we need to have incoming signals in phase!
- The geometric delay is an example of a correction.
- Various other effects can make our signals become out of phase resulting in no signal we need to correct for this.

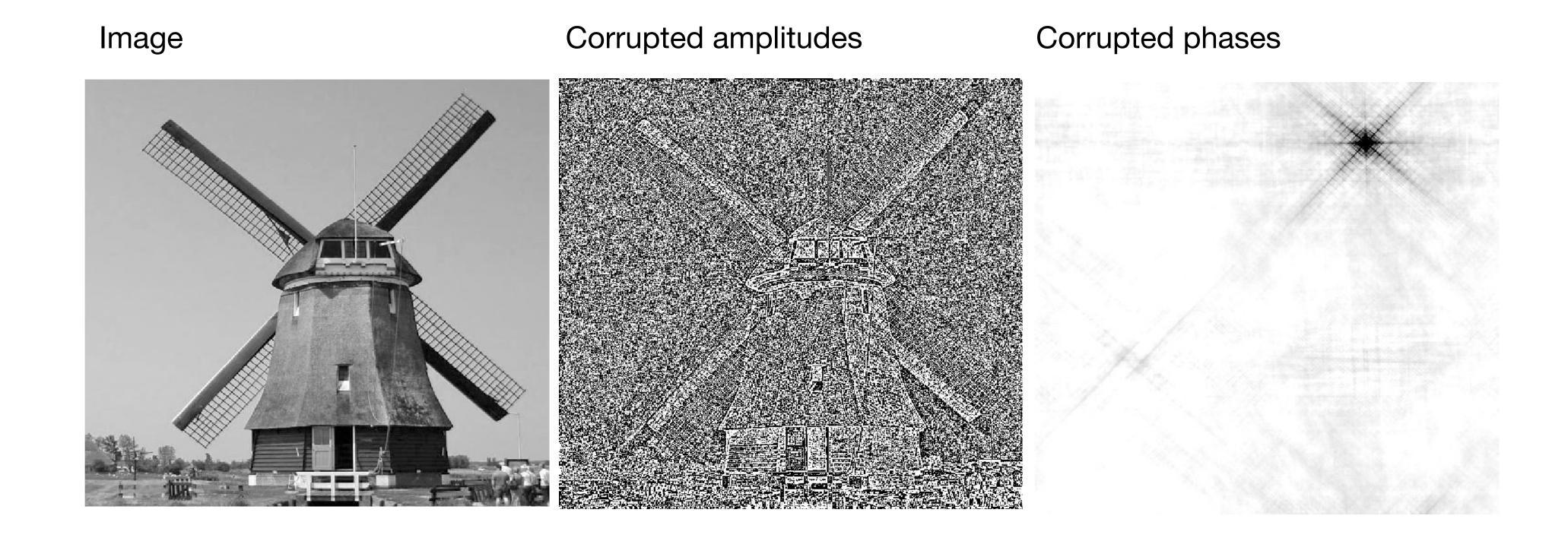
• In addition, we assume that the source flux entering each antenna is constant - there are also

corruption effects that can change this too!



Corruption of interferometric data

- As we measure the Fourier plane with our interferometer, not correcting for these errors can have drastic effects.
- Phase encodes position, amplitude encodes spatial frequency power!



Key points for calibration

- For an interferometer to work well we need to have:
 - Phases aligned for constructive interference
 - Amplitudes need to be constant we assume that the sky brightness distribution, B, is constant (in flux and position)!
 - A flux scale (similar to temperature scale) i.e., how bright is your source relative to something of some known (physical) brightness.
 - Bad data removed any time the telescope isn't looking at a source, or there is interference (from mobiles) needs to be removed.
- And remember that we need to do this with respect to time & frequency on all baselines!

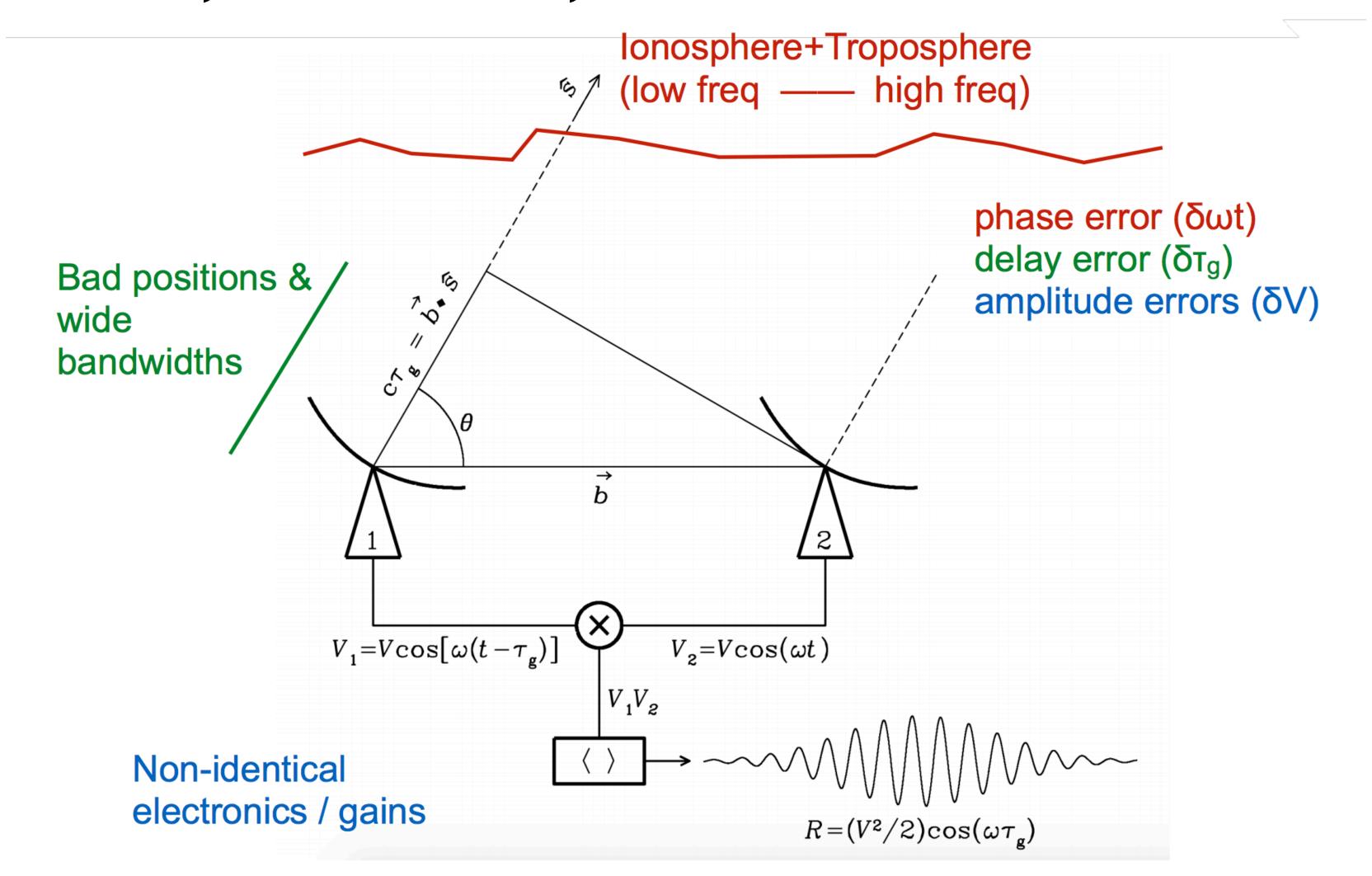
Calibration is merely removing the corrupting effects.

Some key terms

Over the course of this lecture there will be some nomenclature used which you should try to remember.

- Complex gains this is both amplitude and phases
- Bandwidth range of frequencies being measured by the interferometer
 - Spectral window sub-bands in frequency determined by the receiver system
 - Channel sub-bands of spectral windows
- **Delays** this is the phase error derivative with respect to frequency i.e., $\mathrm{d}\phi/\mathrm{d}\nu$
- **Phases** this is the phase error with respect to time and frequency i.e., $\phi(t,\nu)$
- Rates this is the phase error derivative with respect to time i.e., $d\phi/dt$
- Flagging the removal of bad data e.g., affected by radio interference

So what can, and will, attack our data?



Solve for these issues using calibration

Sources of errors i.e. what attacks our data

Atmosphere

- lonosphere
- Troposphere
- Water vapour

LNA / conversion chain

- Clock
- Gain, phase, delay
- Frequency response

Antenna / feed

- System temperature
- Primary beam
- Pointing
- Antenna location

Digitiser / Correlator

- Auto-leveling
- Baseline errors

Radio Frequency Interference (RFI) 2. Error sources

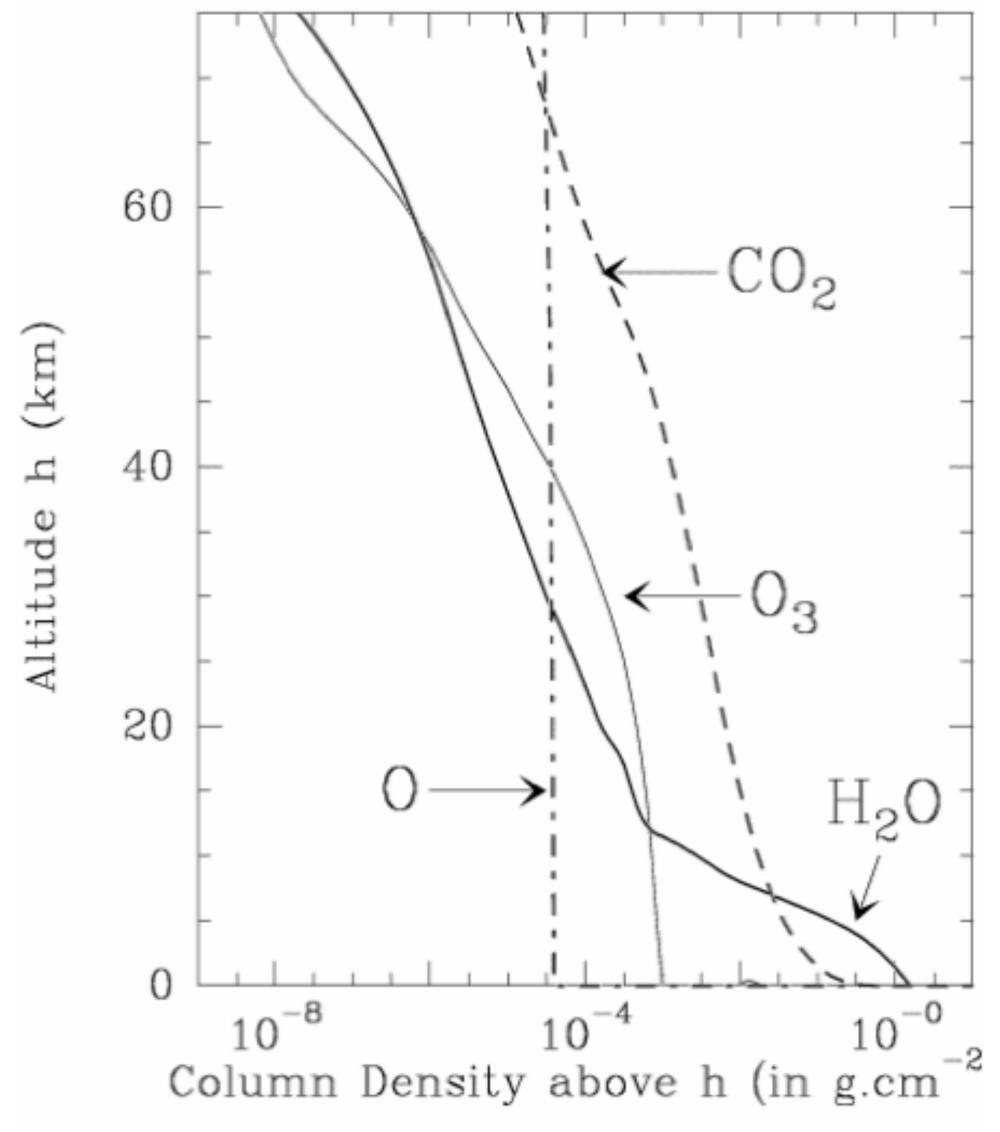
The troposphere

- Molecular refraction
 - 'Wet' H2O vapour (Clouds worse)!
 - 'Dry' e.g. O2, O3
- Refracts radio waves
- Phase distorted

$$-\phi = \frac{n_{\rm w} 2\pi}{\lambda}$$

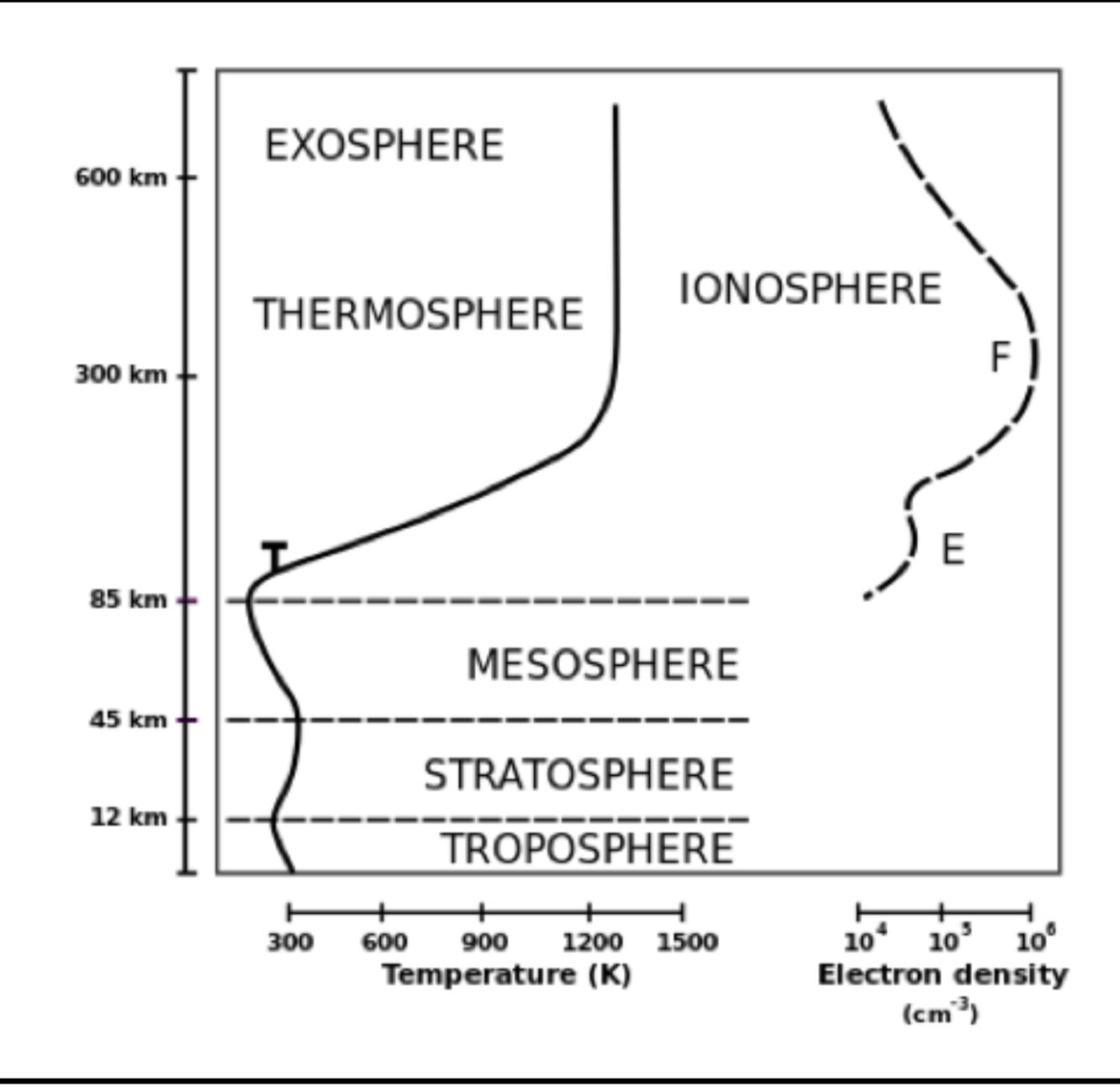
- $n_{\rm w}$ is water vapour refractive index
- Tropospheric errors $\propto 1/\lambda$
 - Significant at high frequencies $\nu > 15\,\mathrm{GHz}$
 - Sub-mm observing at cold, high, dry sites





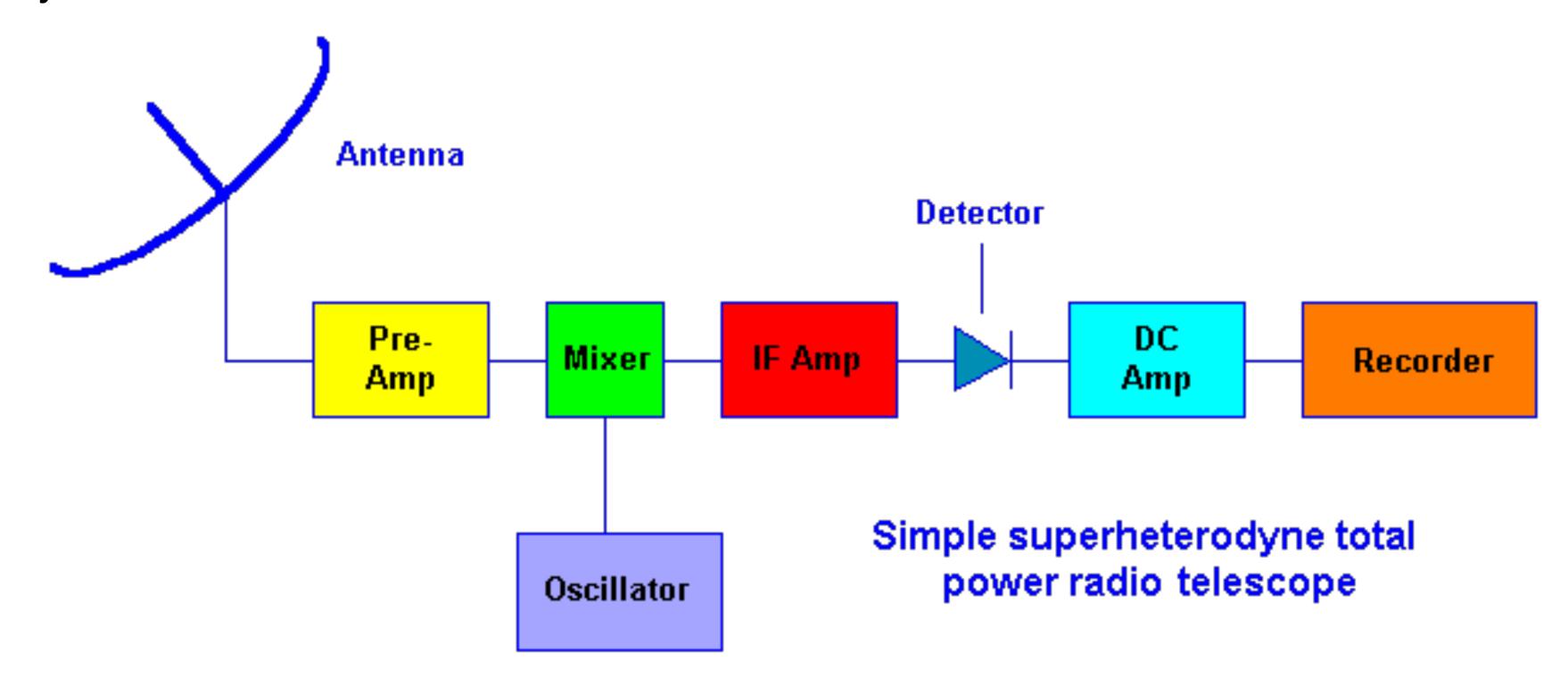
The ionosphere

- Refraction by electrons
- Delay $\mathrm{d}\phi/\mathrm{d}\nu \propto N_e \lambda^2$
 - N_e = atmospheric electron column density
- Electrons spiral round Earth's magnetic field
 - Polarisation angle of radiation Faraday rotated
- Ionospheric errors worst at $\nu < 1~\mathrm{GHz}$
 - Exacerbated by solar activity
 - $\lambda > 20 \, \mathrm{m}$ only from space



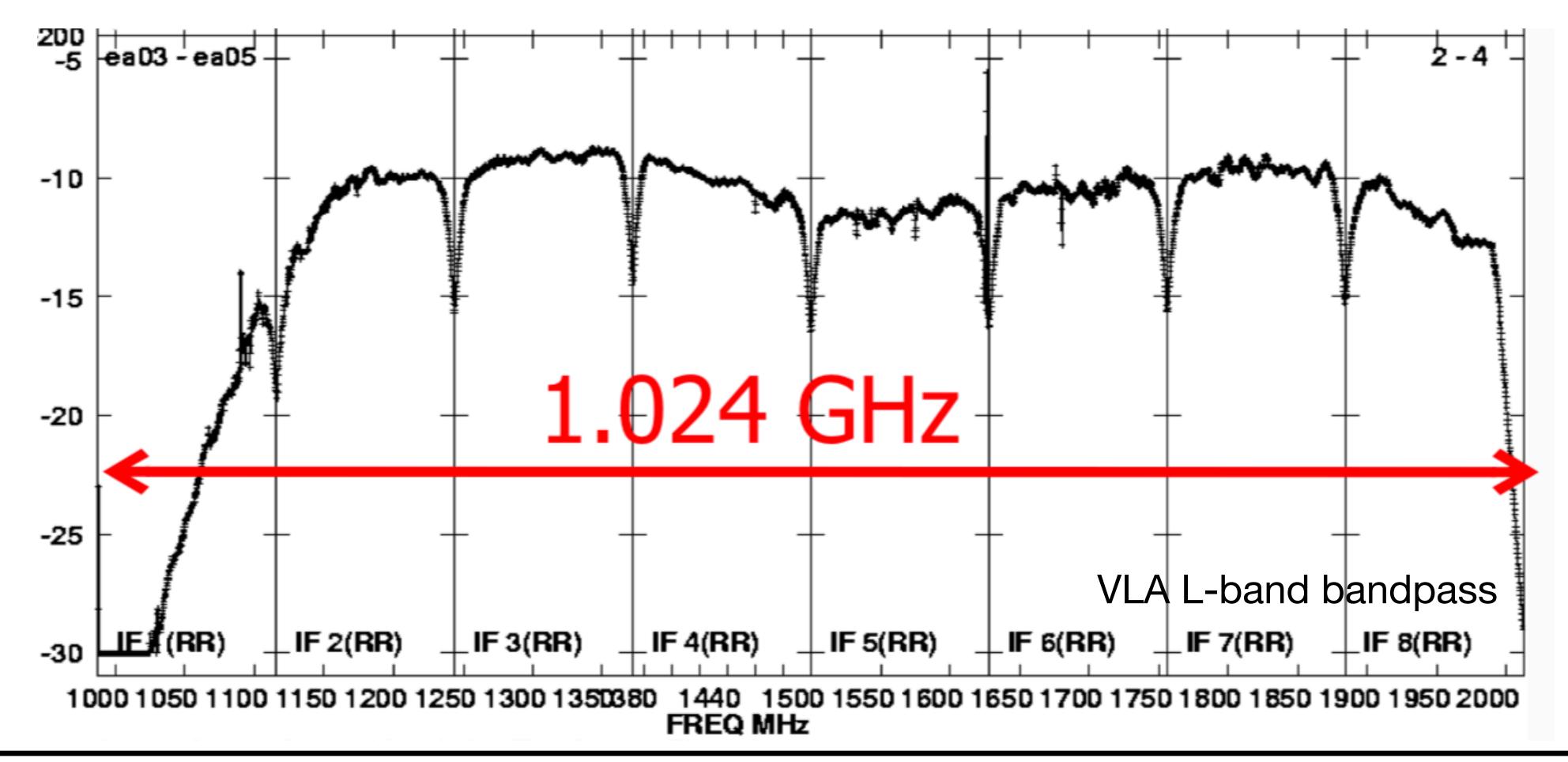
At the telescope - gain variations & delay chains

- Signals at antenna need to be amplified but LNA has variable gains corrupts amplitude
- Signals also pass through different electronics at different frequencies corrupts phase vs frequency



At the telescope - bandpass

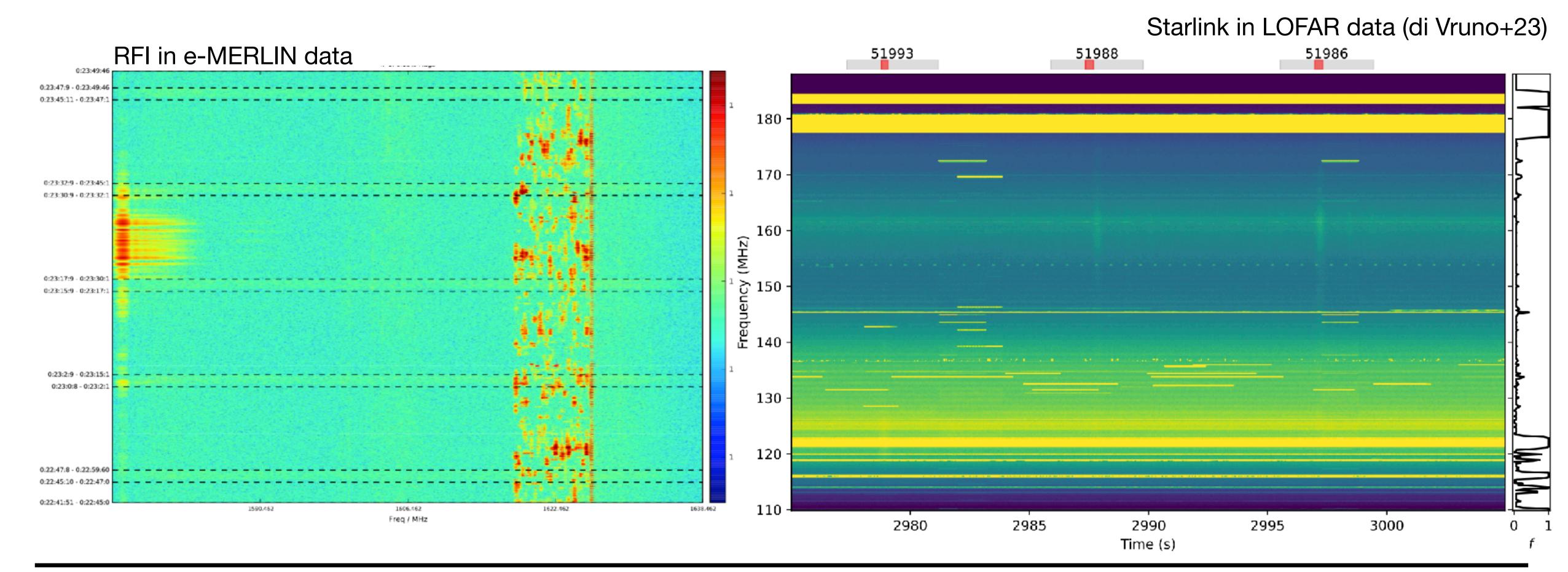
Antenna receivers and filters gives the sensitivity of each antenna vs frequency a
distinctive shape called the bandpass. This needs to be corrected and flattened.



2. Error sources

Radio frequency interference (RFI)

- Your mobile phone is brighter than any radio source in the sky...
- Causes errors, so this needs to be removed from our data! see the flagging slide deck.



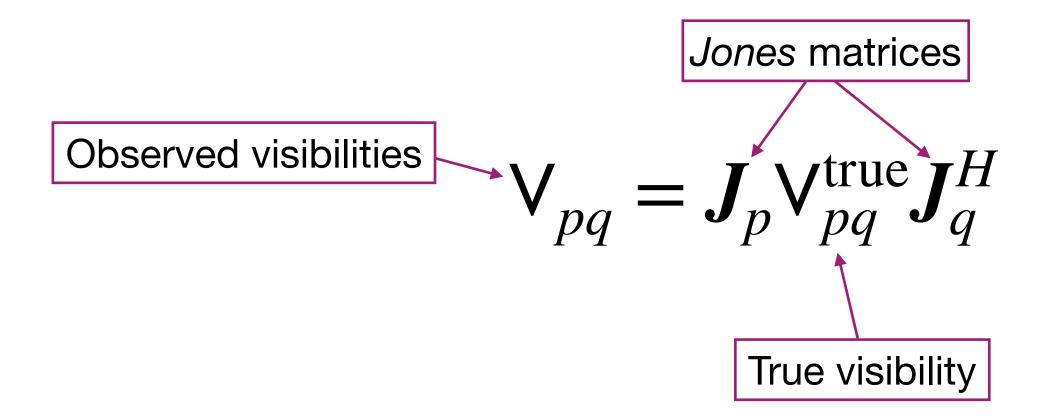
Key points for calibration

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 - Phases aligned for constructive interference
 - Amplitudes need to be constant we assume that the sky brightness distribution, B, is constant (in flux and position)!
 - A flux scale (similar to temperature scale) i.e., how bright is your source relative to something of some known (physical) brightness.
 - Bad data removed any time the telescope isn't looking at a source, or there is interference (from mobiles) needs to be removed.
- And remember that we need to do this with respect to time & frequency on all baselines!

Calibration is merely removing the corrupting effects.

Parameterising calibration

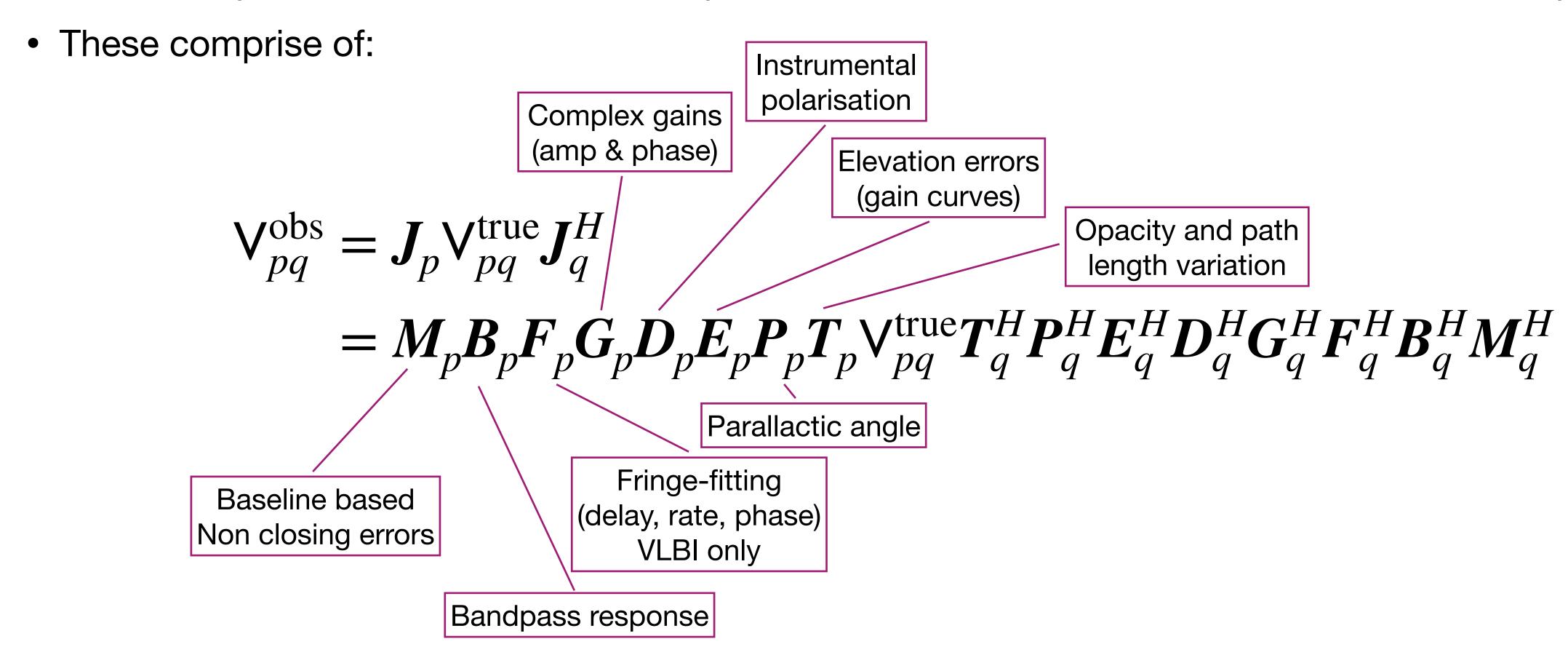
- We want to parameterise our knowledge of the system as some quantities need to be derived (e.g. phase, delays, amplitudes)
- We use the radio interferometry measurement equation (RIME) to do this, which relates the observed (perturbed) visibility to the 'real'/ ideal (unperturbed) visibility i.e.:



- The Jones matrices encodes everything that "happens" to the signal from the source to correlator.
- This assumes calibration parameters should be antenna-based.

Decomposing the corruption effects

• We decompose the RIME calibration equation into different terms which are solved for independently.



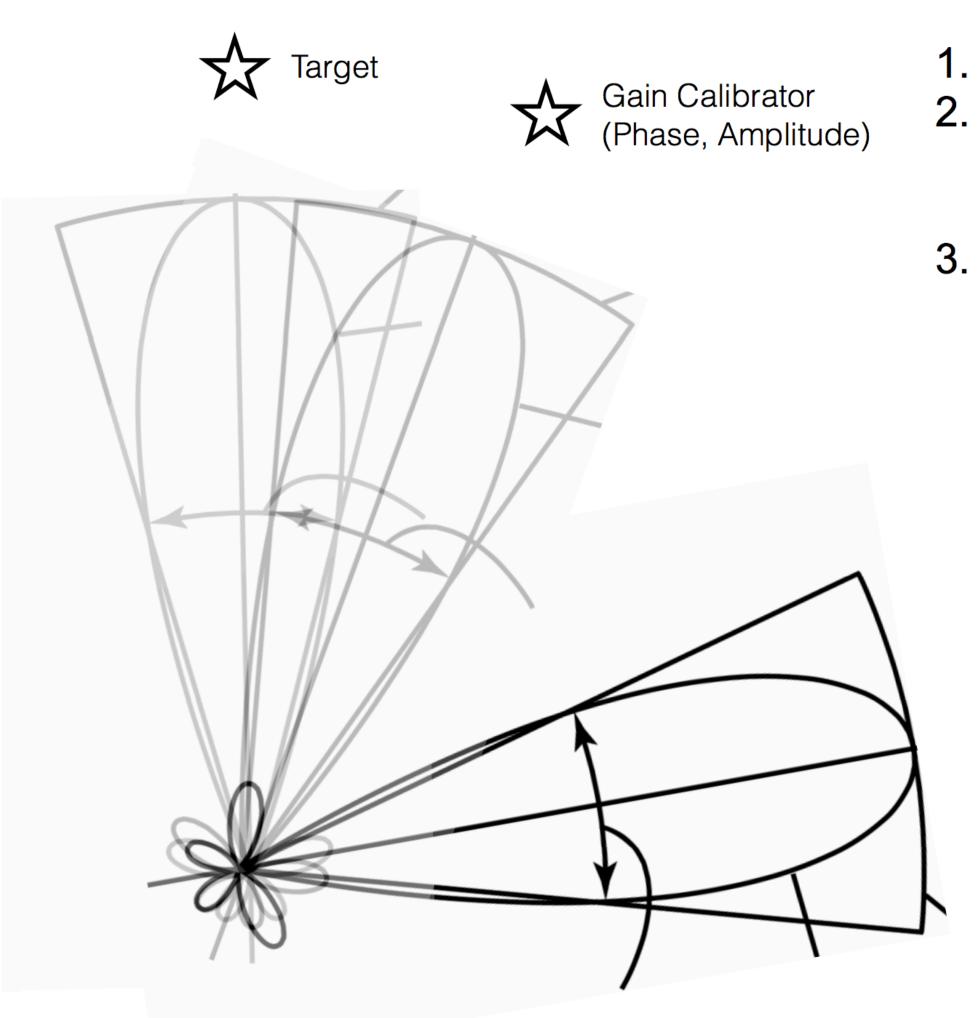
Observational set-up

- To be able to mediate these corrupting effects we need to set up our observations such that we can correct for all of these.
- To be able to do this we normally split our calibration products into certain categories
 - a priori calibration these are calibration products that are either given to you, or are corrected for by the radio observatory e.g. the geometric delays!
 - flux scaling setting the 'temperature' scale of your observations.
 - source-based calibration & phase referencing the observational set-up allows you to calibrate for complex gains (i.e. amplitude and phase) using a bright source e.g. removing the atmospheric contributions.

On-source calibration

- However, the on-source calibration can be split into two main terms:
 - **Time-independent terms** i.e. the errors occur on long timescales > observation time and are often located at the antenna so are also direction-independent (bandpass is one example).
 - **Time-dependent terms** the errors change on quick timescales and often do care about the pointing direction e.g. residual geometric delay errors, atmospheric phase shifts, LNA gain variations
- For the time independent terms, the observatory can measure these and give them to you a priori OR you observe a very bright calibrator.
- For the time-dependent terms, we often use a useful FT relation to calibrate our phases and amplitudes that of a **point source!**

A typical VLBI set up



- 1. Observe source
- 2. Observe calibrator to measure gains (amplitude and phase) as a function of time.
- 3. Observe **bright calibrator** of known flux-density and spectrum to measure absolute flux calibration, band-pass and residual delays

Flux Calibrator / fringe finders (Flux, Bandpass, Delay)

A priori calibration

- A priori calibration concerns calibration done either by the observatory or given to you once you receive the data.
- The following effects are often calibrated by the observatory
 - 1. Delay tracking
 - Correctable off-line if within Nyquist or sensitivity limit
 - Phase tones can be used to align antenna signals
 - 2. Antenna positions
 - Errors cause bad delays
 - Cannot transfer phase-ref corrections accurately to target
 - 3. Geometric delay correction
 - Done during correlation using accurate antenna locations
- You still need to apply with information given by observatory:
 - Gain curves antenna efficiency vs source elevation (explained next)
 - $T_{
 m sys}$ for flux scaling (explained later)

A priori calibration - gain curves

$$V_{pq}^{\text{obs}} = M_p B_p F_p G_p D_p E_p P_p T_p V_{pq}^{\text{true}}...$$

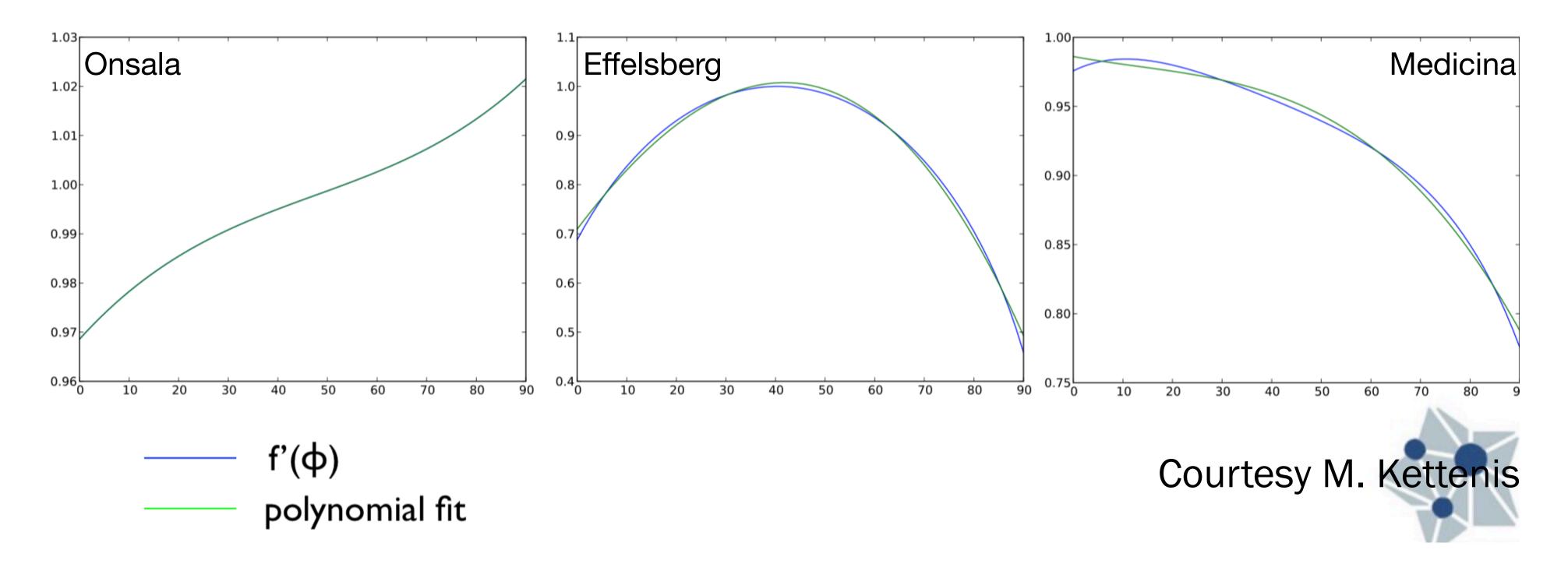
Atmosphere adds noise and absorbs signal

$$T_{\text{received}} = T_{\text{source}} \exp\left(\frac{\tau_{\text{atm}}}{\cos(z)}\right) + T_{\text{atm}} \left[1 - \exp\left(\frac{\tau_{\text{atm}}}{\cos(z)}\right)\right]$$

- Source would provide temperature T if measured above the atmosphere optical depth $au_{
 m atm}$ and z is the zenith distance.
- Noise is increased for observing at low elevation (large z)
- We normally apply an analytic gain curve (assume $au_{
 m atm}$ stable)

A priori calibration - gain curves

- As well as correcting for the atmospheric noise, antennas are not rigid
- → their effective collecting area and net surface accuracy vary with elevation as gravity deforms the surface.
- More important at higher frequencies



A priori calibration

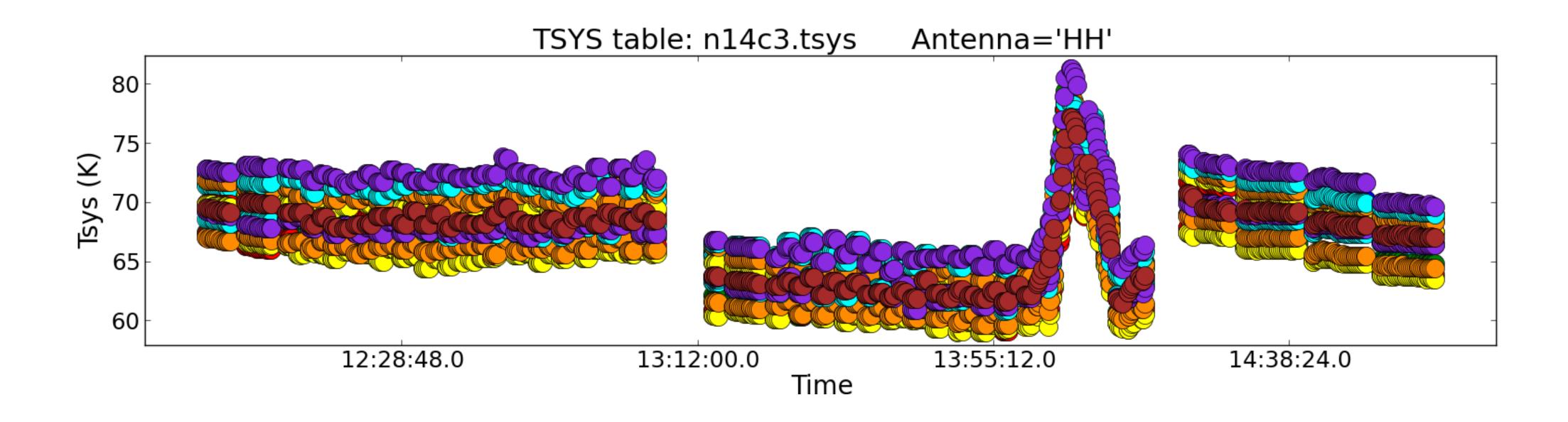
- Calibration measurements supplied with data can includes Tsys, gain-elevation and WVR
- Water Vapour Radiometry (at mm/sub-mm wavelengths): measure atmospheric water line every few seconds, calculate refractive delay of phase and/or absorption.
- Antenna position corrections may also be available.
- For VLBI and low frequency, ionospheric total electron content measures can be used to correct dispersive delays (i.e. curvature of delay term across band)
- Others include weather tables to refine gain-el; GPS measurements for position and Faraday rotation.
- May need reformatting or removal of bad values
 - Usually employing standard scripts, often by observatory staff

Flux scaling

- The visibility amplitudes that come out of the correlator have some arbitrary scaling.
- Flux scaling is concerned with converting these amplitudes to physically meaningful units i.e. Janskys
- To do this, you need to observe something with a physically known flux density.
- This is normally either:
 - A standard source e.g. 3C84 known as bootstrapping (not typical for VLBI observations)
 - Or by reference to a noise diode (of known brightness) on the antenna known as a $T_{\rm svs}$ measurement (remember this value encodes the antenna sensitivity)

Flux scaling - Tsys

- The normal method for VLBI is to observe a noise diode with known flux density every few minutes.
- This gives you a measure of your telescope system 'temperature', $T_{
 m sys}$. You can then work out what the source flux density is relative to that.



Phase referencing

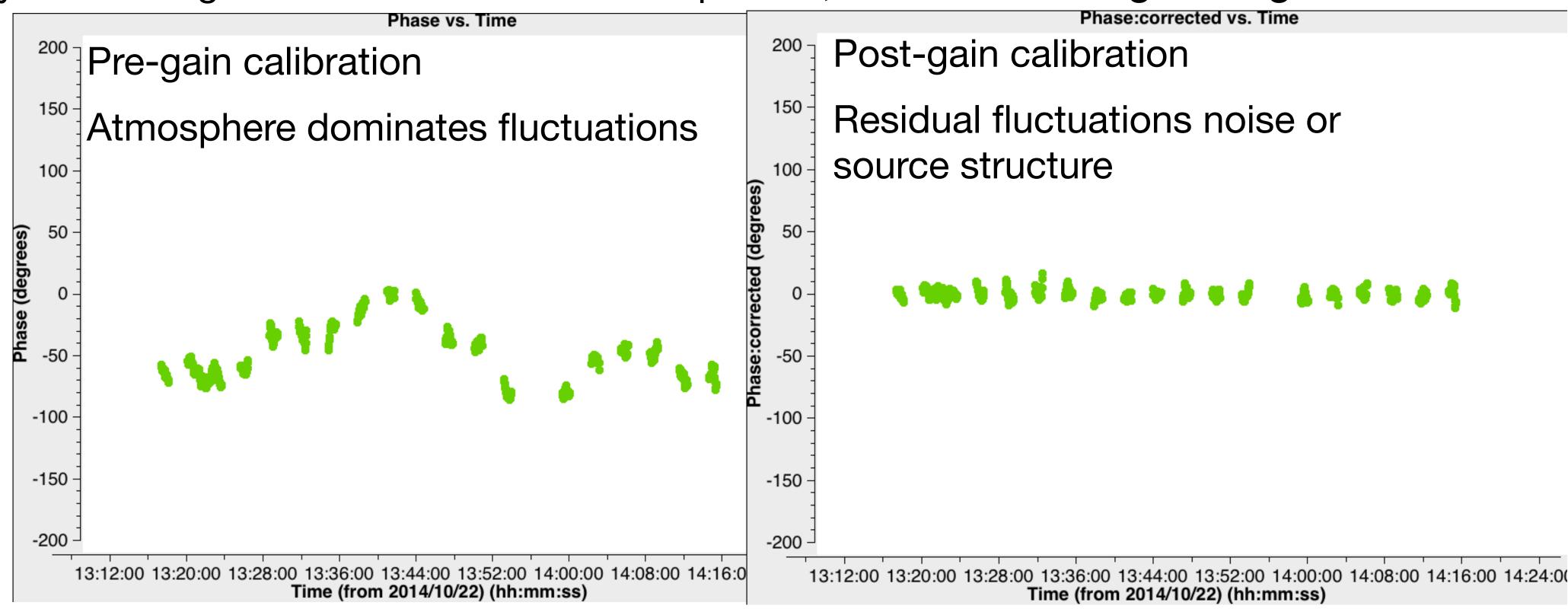
$$V_{pq}^{\text{obs}} = M_p B_p F_p G_p D_p E_p P_p T_p V_{pq}^{\text{true}}...$$

- Your target / science target has unknown structure but we can use a point source to remove the effects of the atmosphere (corrupting phases) & the antenna gains (corrupting the amplitudes) by simply comparing the differences between a point source in FT space and observed visibilities
- The atmosphere is not the same everywhere so we need the point source to be near the target field.
- Also the atmosphere changes rapidly (~ 5 min at 1.4 GHz) so we need to track these variations.



Phase referencing

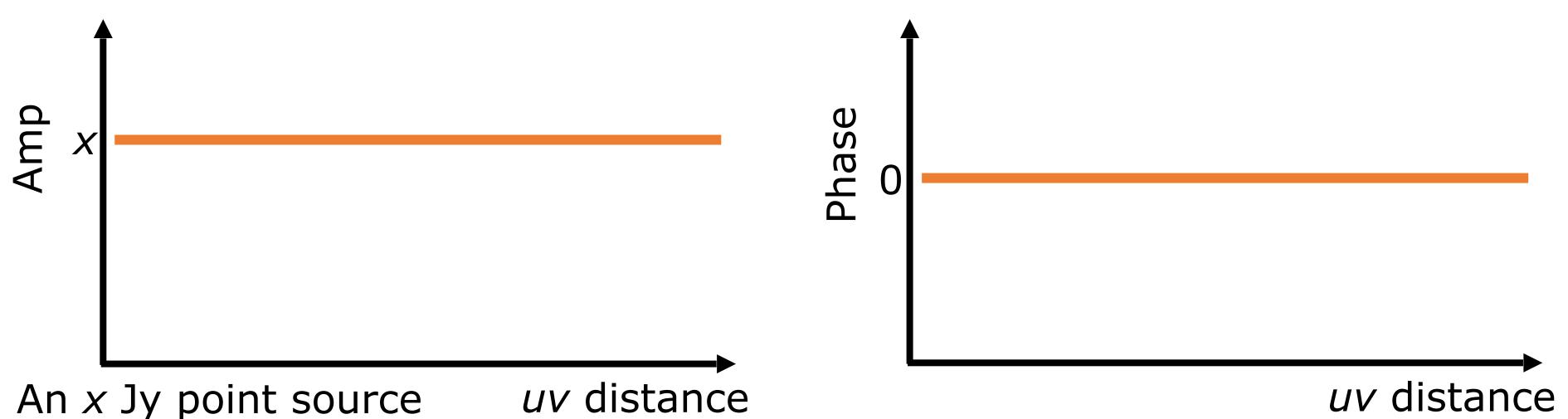
- Therefore, to do this we have to move between calibrator (known as the phase calibrator) and our target field often (cycle time ~ 10 mins at 1.4 GHz, more frequent at higher frequencies)
- We can calibrate phases & delays (phase vs frequency). For VLBI, it's a little more complicated and you need higher order terms to solve for phases, this is called **fringe fitting**.



Important aside

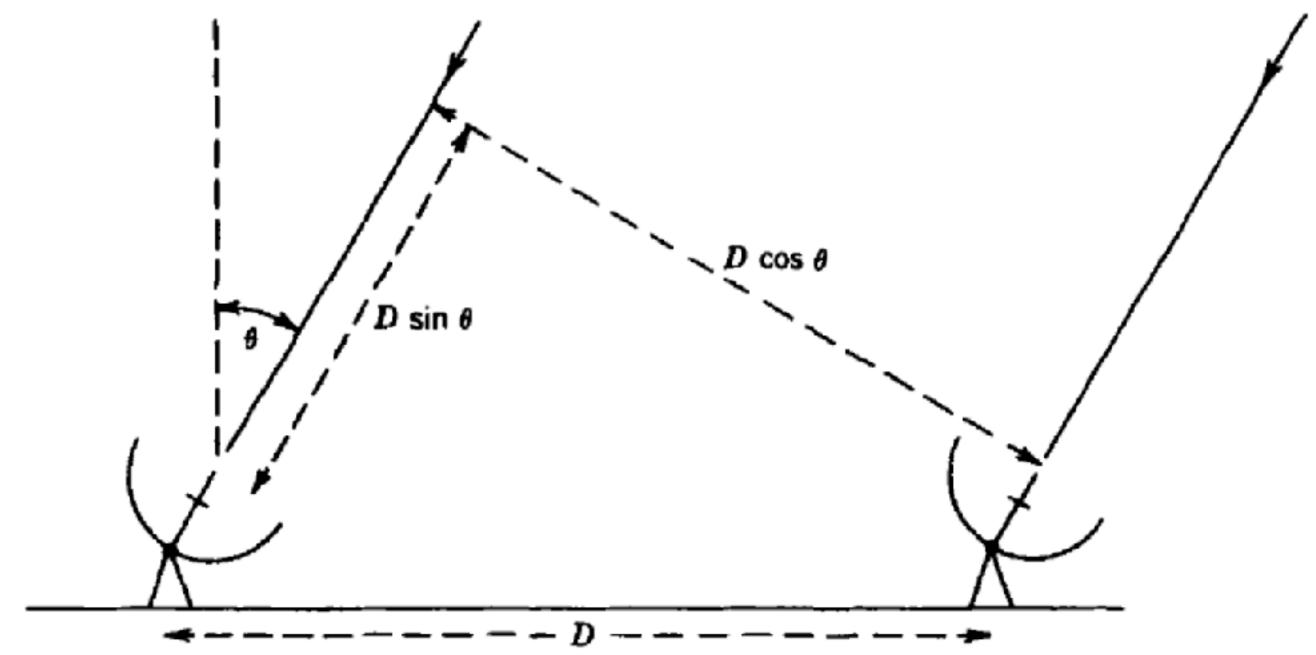
Point sources

- Lots of calibration assumes that your phase calibrator is point-like and in the centre of the field (i.e. phase center).
- Calibration essentially compares your model (i.e. point source) with the observed visibilities and derives corrections.
- A true point source is flat in amplitude and phase space (see below)
- If your phase calibrator is **not** point-like then we need to derive a model. We will learn more about this
 in the self-calibration lecture.



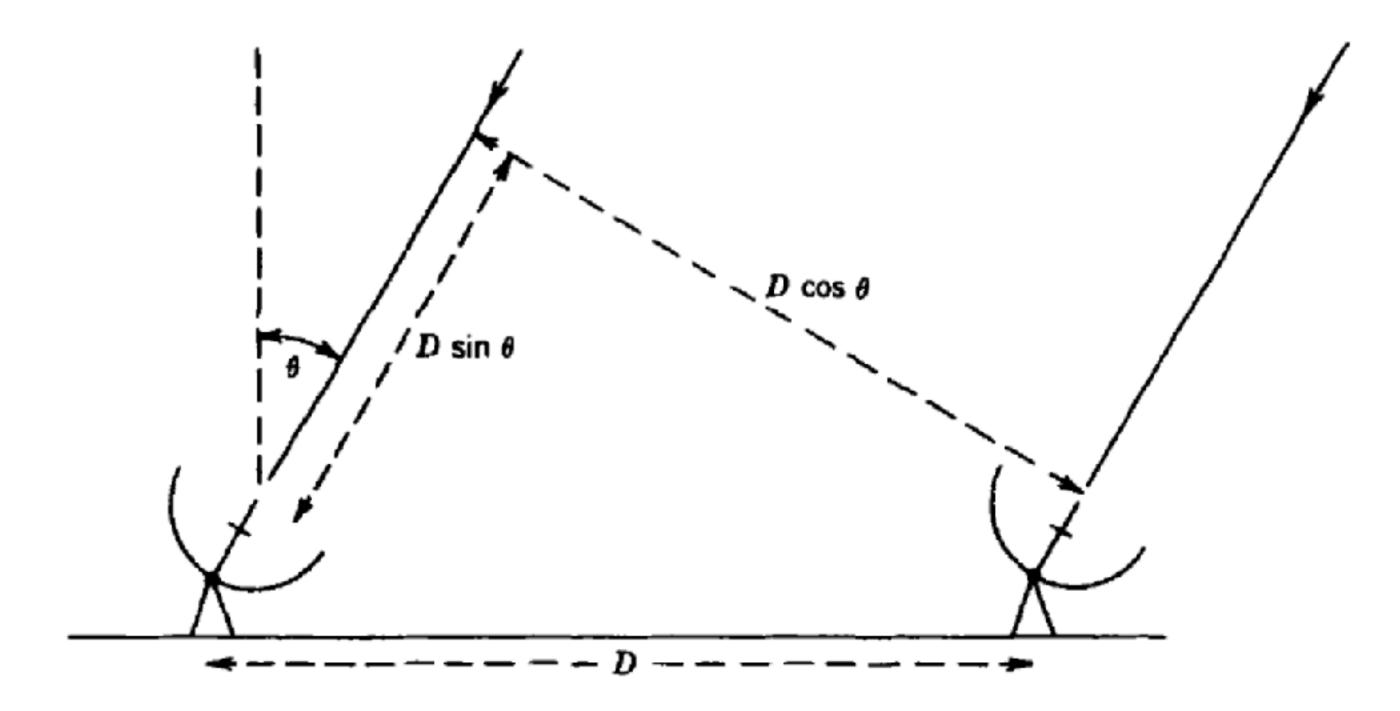
Fringe fitting - introduction

• Recall the simple 2-element interferometer



- Wave-fronts of a signal from a distant source, arrives at one antenna with a geometrical delay, $\tau_{\rm obs} = (D/c)\sin(\theta)$
- Phase difference 'interferometer phase', $\phi = 2\pi\nu\tau_{\rm obs}$, changes with time!

Fringe fitting - introduction



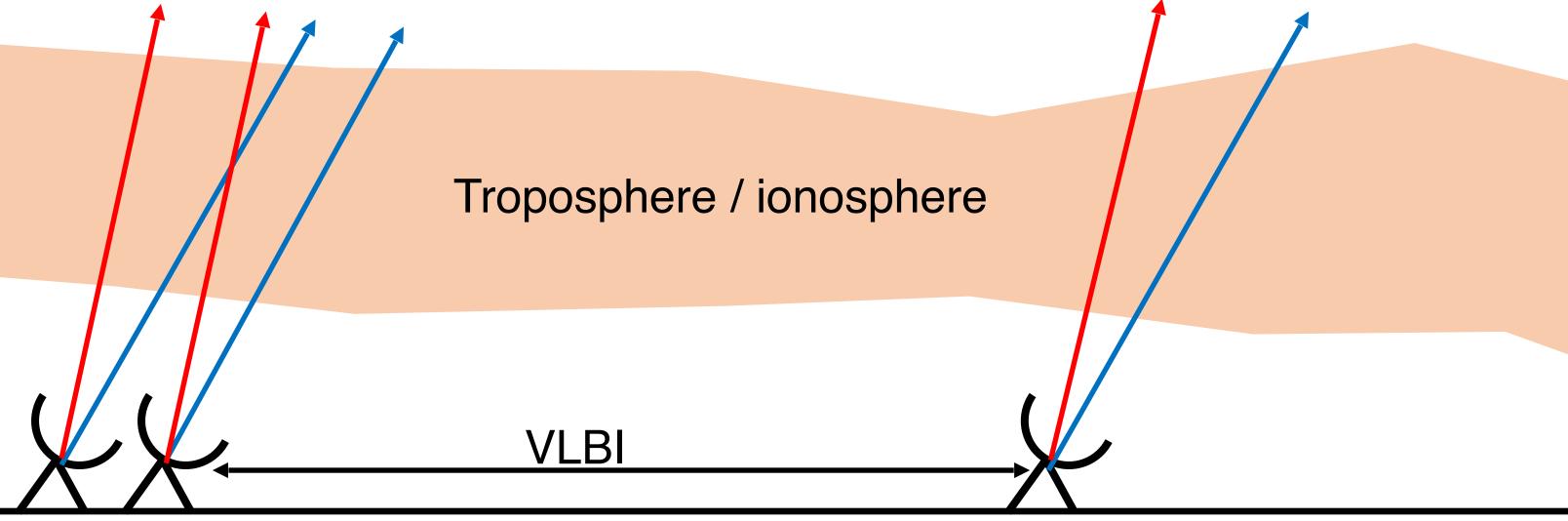
- Signals from both antennas are combined in a correlator
- Correlator estimates and corrects for geometric delays
- For connected arrays e.g. JVLA, ATCA, MeerKAT this simple geometrical delay is enough ... not so for VLBI.

7. Fringe fitting

Why we need fringe-fitting

VLBI vs short baseline arrays

- No fundamental difference but with longer baselines (100's to 1000's km)
- However VLBI arrays are not connected so:
 - Independent clocks and equipment → phase/delay errors
 - The delay and rate of the wavefronts vary more rapidly due to completely different atmospheric paths.
 - Geometric delay needs to be exact must be estimated and removed during correlation



Why we need fringe-fitting

The geometric model

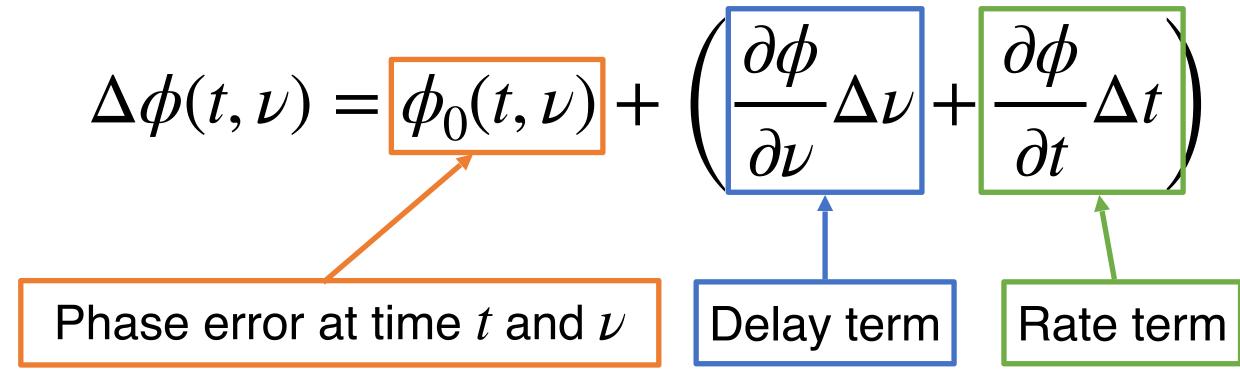
Table 22-1. Terms of a VLBI Geometric Model a

Item	Approx max Magnitude b	Time scale
Zero order geometry.	6000 km	1 day
Nutation	~ 20 "	< 18.6 yr
Precession	$\sim 0.5 \text{ arcmin/yr}$	years
Annual aberration	20"	1 year
Retarded baseline	20 m	1 day
Gravitational delay	$4 \text{ mas } @ 90^{\circ} \text{ from sun}$	1 year
Tectonic motion	10 cm/yr	years
Solid Earth Tide	50 cm	12 hr
Pole Tide	2 cm	$\sim 1 \text{ yr}$
Ocean Loading	$2 \mathrm{~cm}$	12 hr
Atmospheric Loading	$2 \mathrm{~cm}$	weeks
Post-glacial Rebound	several mm/yr	years
Polar motion	0.5"	$\sim 1.2 \text{ years}$
UT1 (Earth rotation)	Random at several mas	Various
Ionosphere	$\sim 2 \text{ m at } 2 \text{ GHz}$	seconds to years
Dry Troposphere	2.3 m at zenith	hours to days
Wet Troposphere	0-30 cm at zenith	seconds to seasonal
Antenna structure	<10 m. 1cm thermal	
Parallactic angle	$0.5 \mathrm{turn}$	hours
Station clocks	few microsec	hours
Source structure	5 cm	years

- Terms that affect the delay > few cm
- Most radio astronomers don't have to worry about these effects
- However, correlator model, not perfect model (due to atmosphere / clock errors)
- Residual phase / delay errors cause decorrelation of signal
- → fringe-fitting solves for this!

How to fringe-fit?

- Need to solve for phase errors in time (rate) and frequency (delay) space
- Remember the interferometer phase: $\phi = 2\pi\nu\tau_{\rm obs}$
- → phase error depends on delay (i.e. against frequency)
- Fringe fitting solves these errors assuming a linear model of the phase error for each antenna i.e.



 Some cases (e.g. space, mm-, low-frequency VLBI) need require higher orders e.g. dispersive delays -

How to fringe-fit?

• Therefore, for each baseline pq this error becomes:

$$\Delta\phi(t,\nu)_{pq} = \phi_{0p} - \phi_{0q} + \left[\left[\frac{\partial\phi_p}{\partial\nu} - \frac{\partial\phi_q}{\partial\nu} \right] \Delta\nu + \left[\frac{\partial\phi_p}{\partial t} - \frac{\partial\phi_q}{\partial t} \right] \Delta t \right].$$

- Fringe-fitting involves solving the above equation, to obtain the errors.
- Via observations of a bright calibrator → phase referencing
 Typically assumes that source is a point source at the phase centre.
- Can be done per baseline or globally (i.e. combine all baselines and derive per antenna)
- Without fringe fitting cannot average in phase and time
- Worse for weaker targets.

How to fringe-fit?

Global fringe fitting

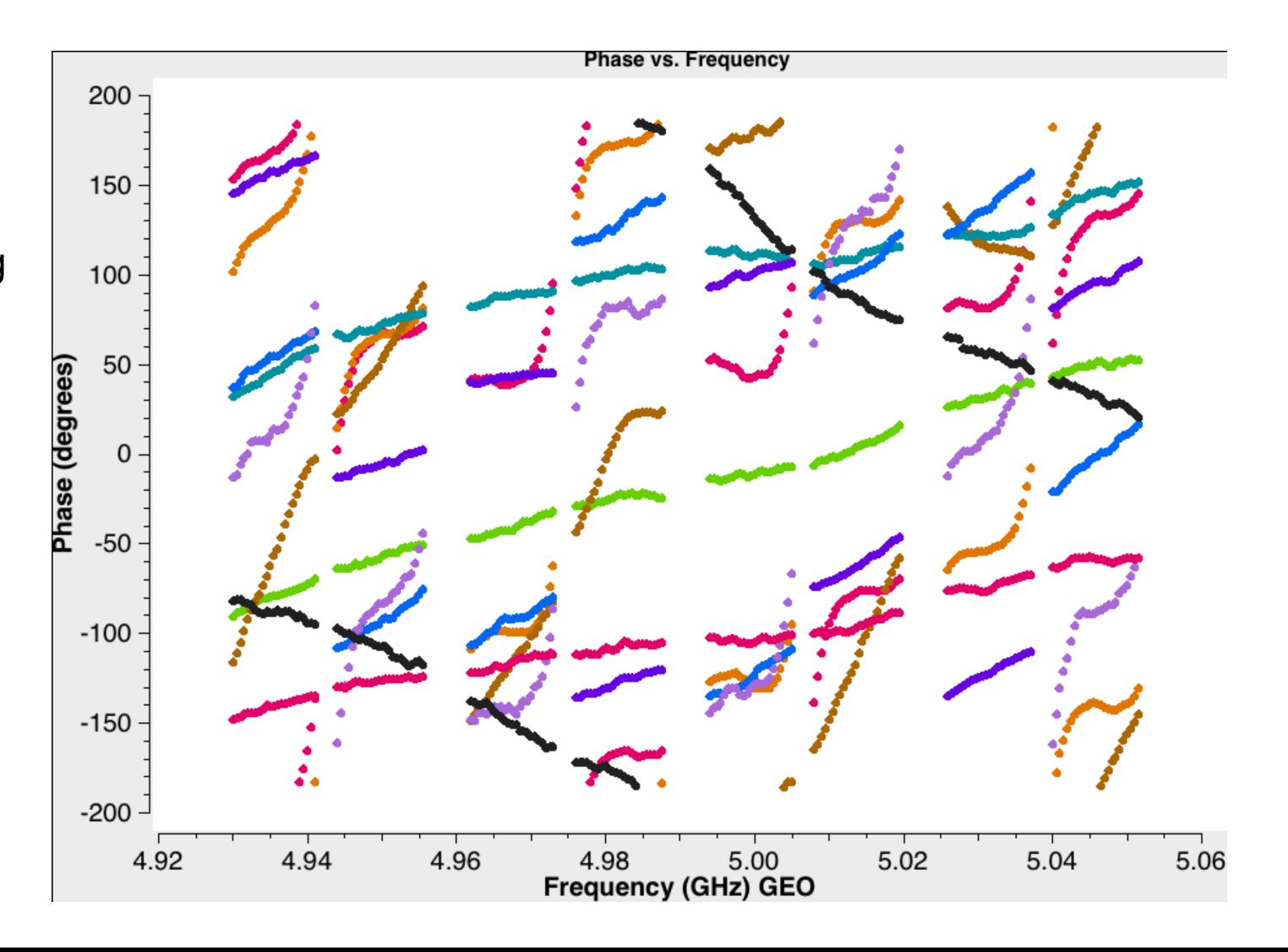
- Use all baselines to jointly estimate the antenna phase, delay and rate relative to a reference antenna
- Solves the baseline phase error equation, with one of the antennas set to the reference antenna
- Delay, rate and phase residuals for reference antenna are set to zero.
- Hence only measures difference, not absolute errors
- Assumes calibrator is a bright point source at phase center (unless model specified!)

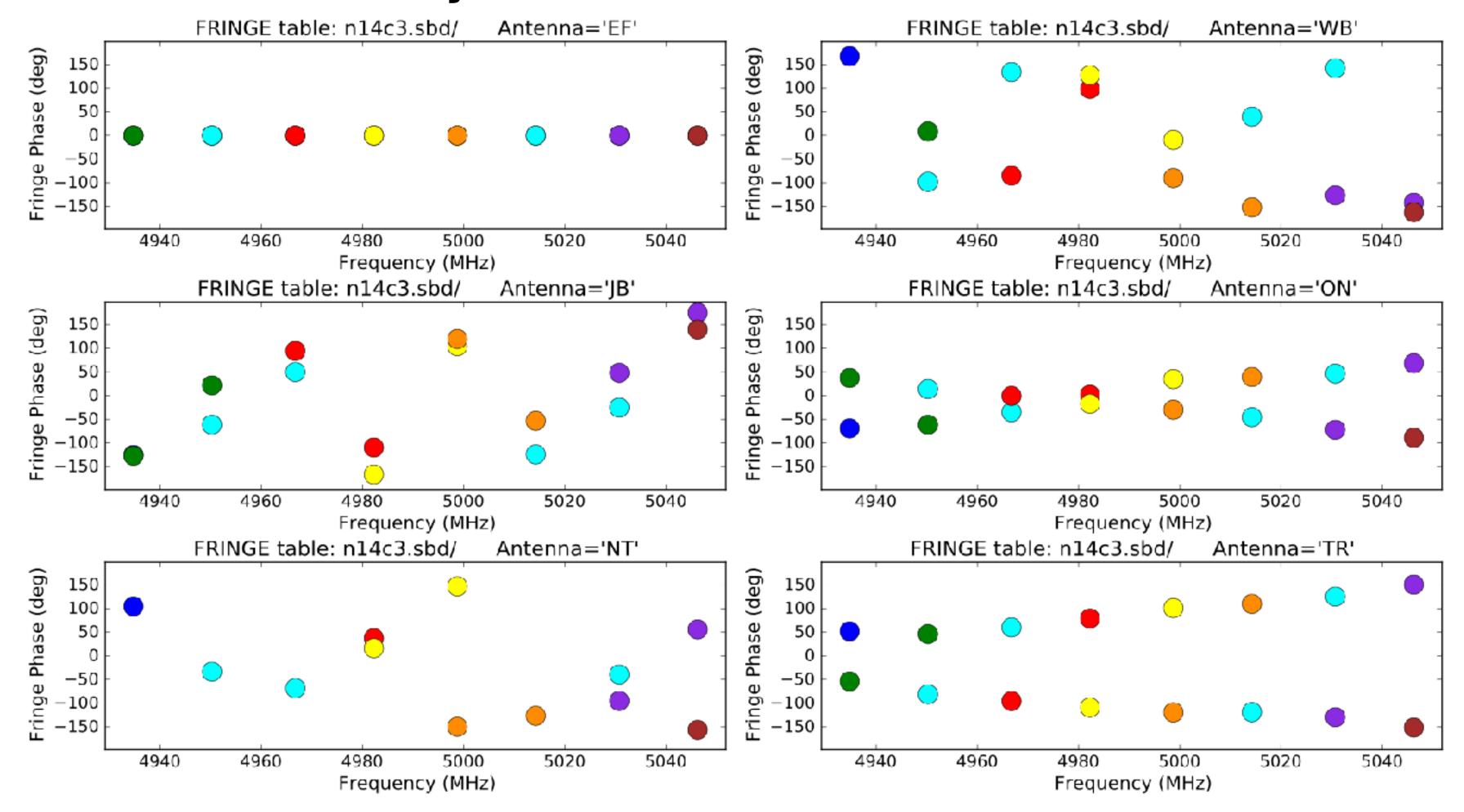
In CASA

- Used to be an AIPS-only task but is now part of CASA (since v 5.3)
- For VLBI, there are (normally) two times we need to fringe-fit.
 - 1. For removing instrumental delays
 - 2. Deriving time, rates and delays variations vs time (known as a multi-band fringe fit)

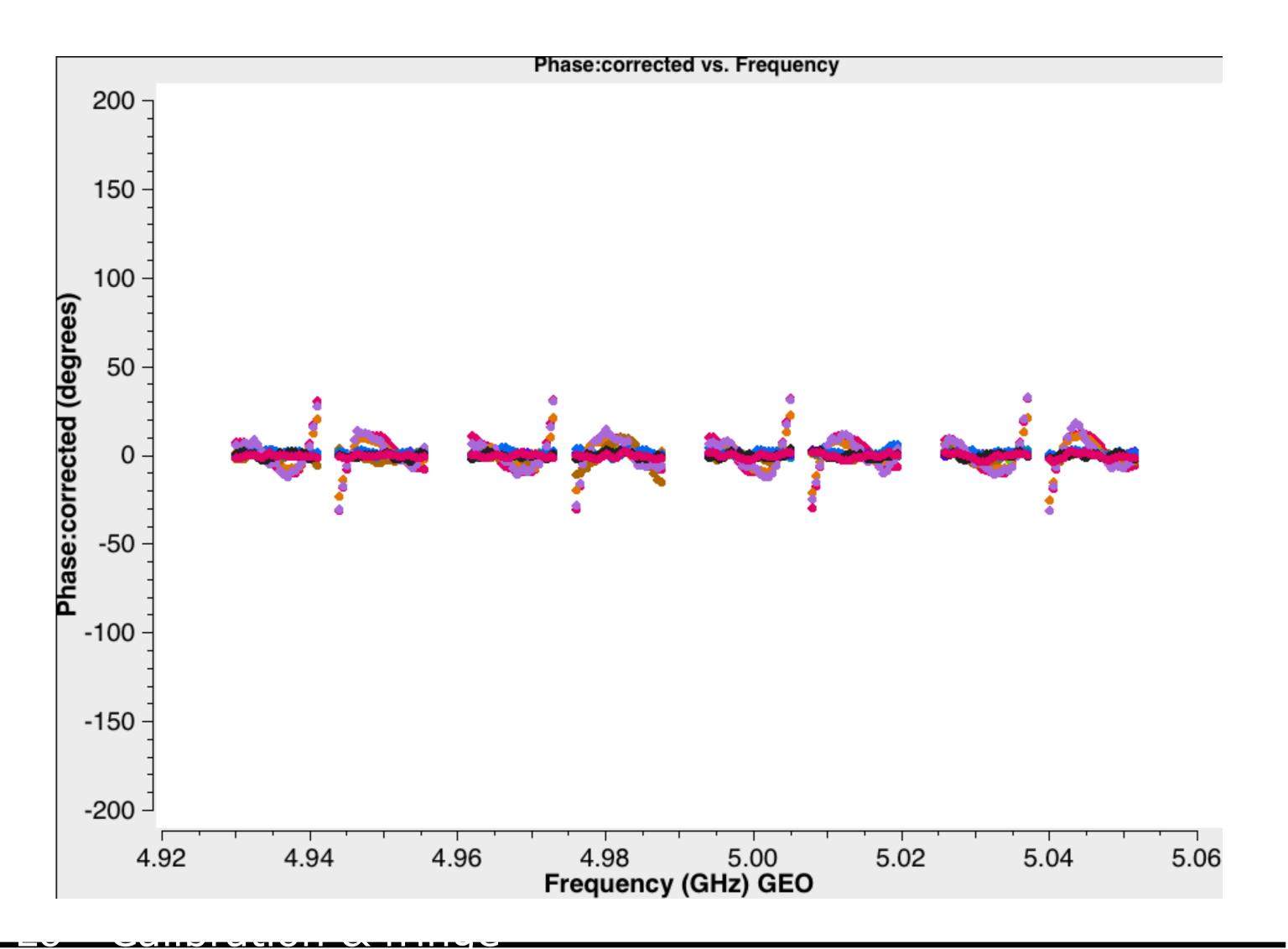
- Typically induced by differing instrumental paths across the receiver subbands (spws)
- Causes 'jumps' in phase across the sub-bands
- Use short integration (~2 mins), on a bright source to get enough S/N per subband.
- Instrumental delays are due to antennas and are not expected to vary across time.

- Before instrumental delay
- Showing phase vs. frequency on bright calibrator (Effelsberg baselines, 1-scan, LL polarisation)
- Coloured by antenna!
- This scan used for deriving solutions.

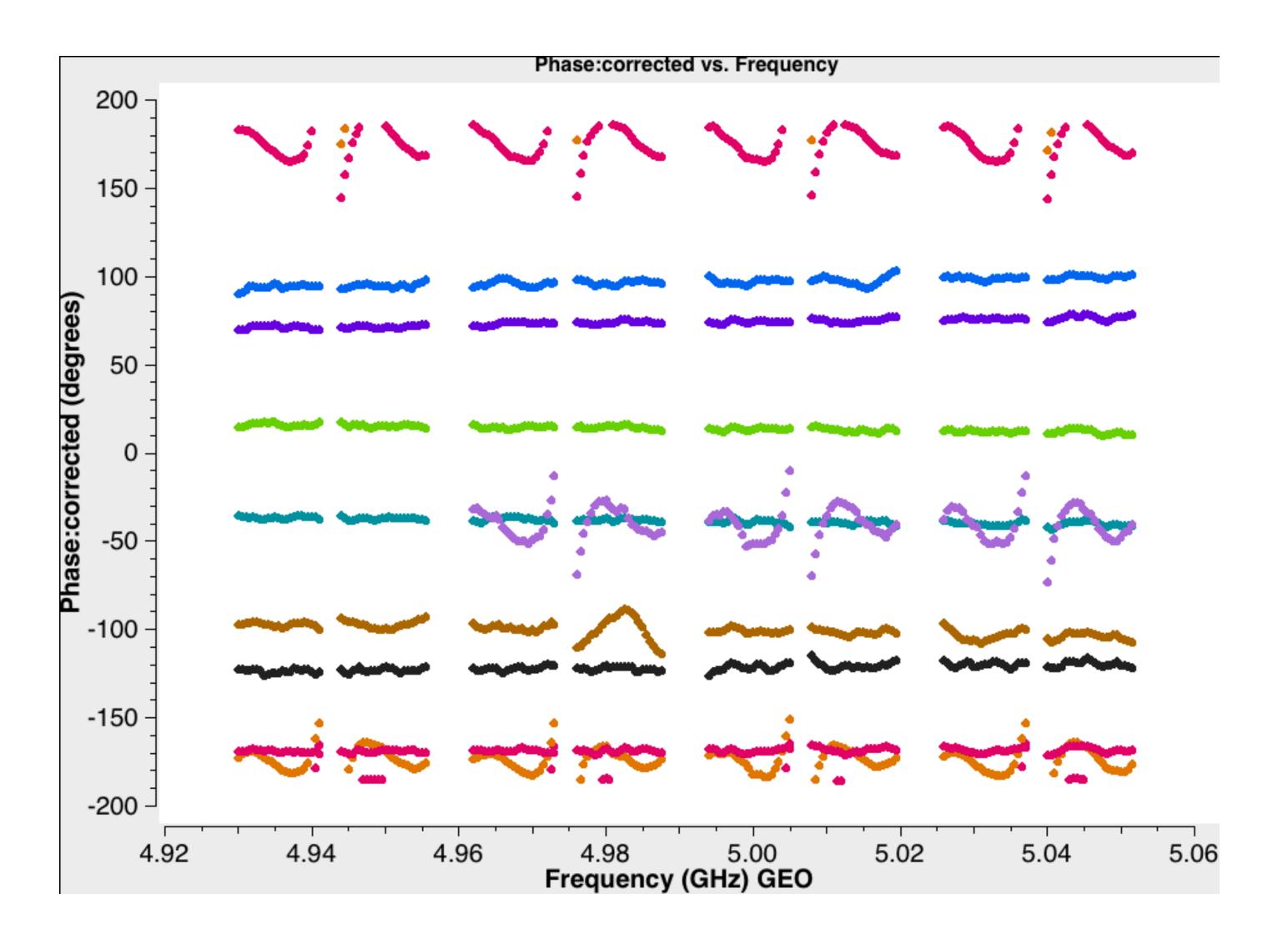




- After instrumental delay
- Showing corrected phase vs. frequency on bright calibrator (Effelsberg baselines, 1-scan, LL polarisation)
- Coloured by antenna!
- Same scan as solutions derived for!



- After instrumental delay
- Showing corrected phase vs. frequency on bright calibrator (Effelsberg baselines, 1-scan, LL polarisation)
- On different scan!
- Phase jumps between subbands gone but time variable remains!

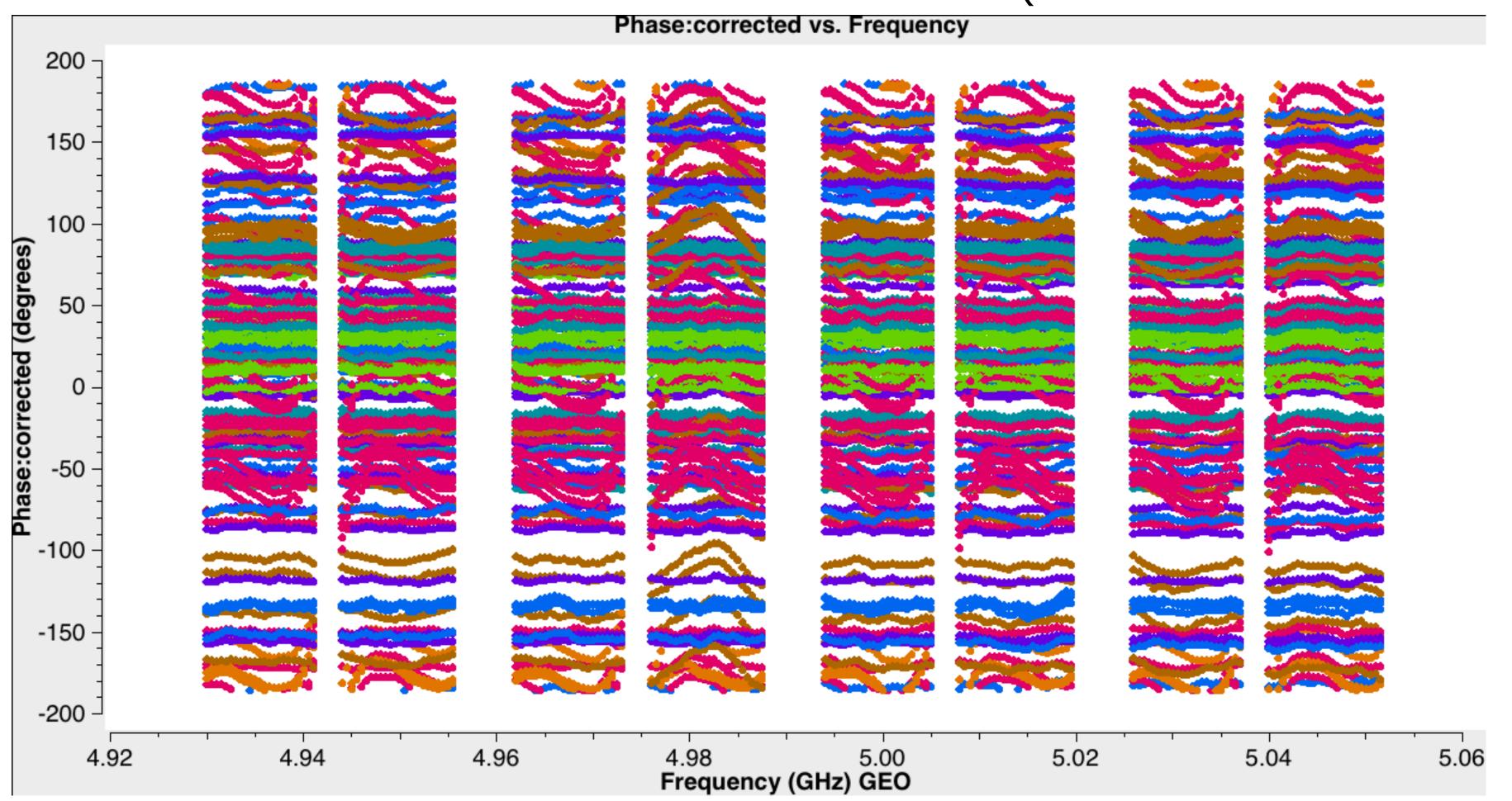


2. Multi-band fringe-fitting

- With instrumental delays removing contributions from the antennas we can expect that the dominant contributor is now the atmosphere.
- This means that any solutions needs to be on the phase calibrator as atmosphere is approximately same as target source
- We want to derive the rate, phase and delays vs time.
- The instrumental delays (time-independent) now allow us to combine the sub-bands together when deriving our time-dependent solutions, therefore phase ref source doesn't need to be so bright!

Multi-band fringe-fitting

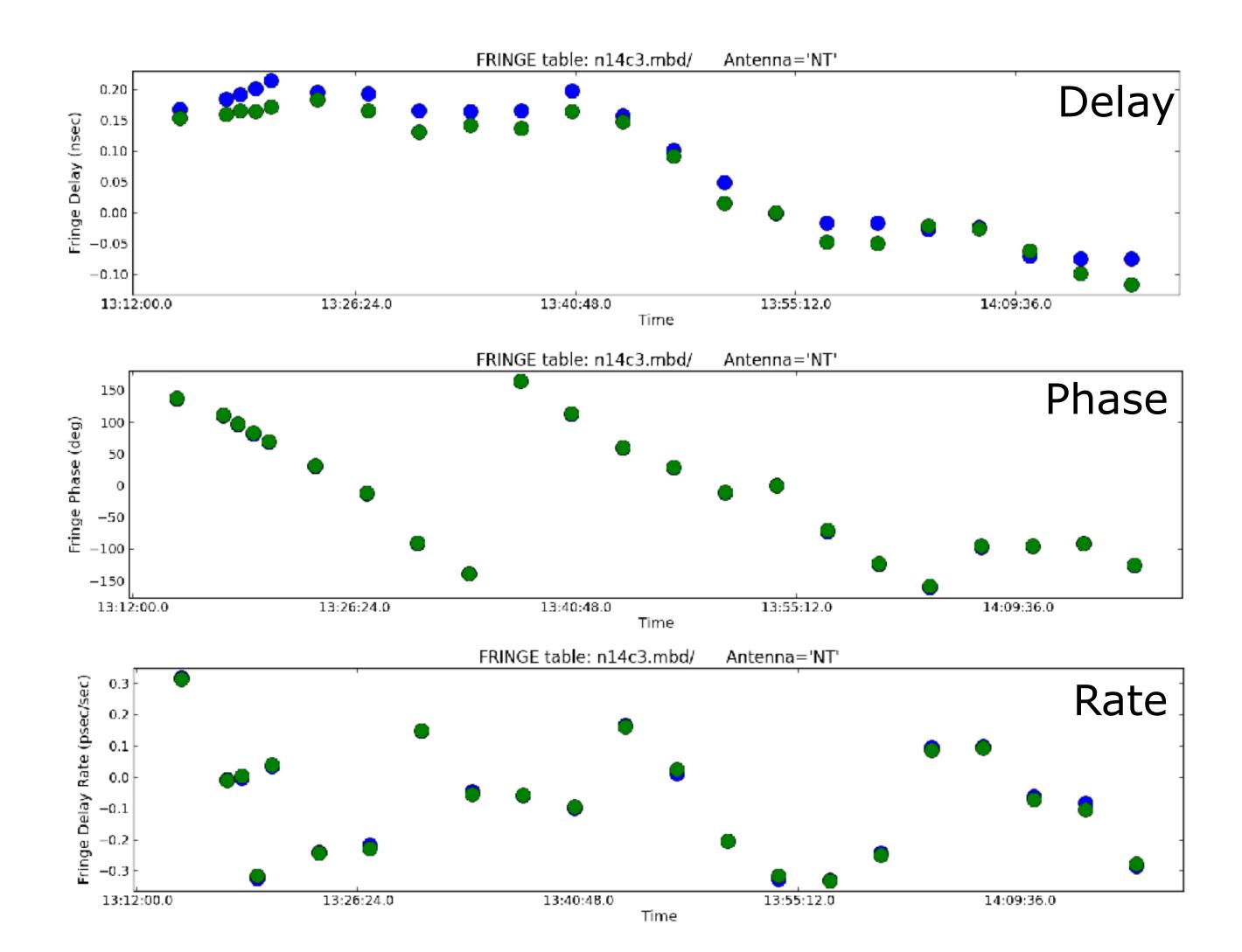
Instrumental delays only (All baselines to Effelsberg)



Multi-band fringe-fitting

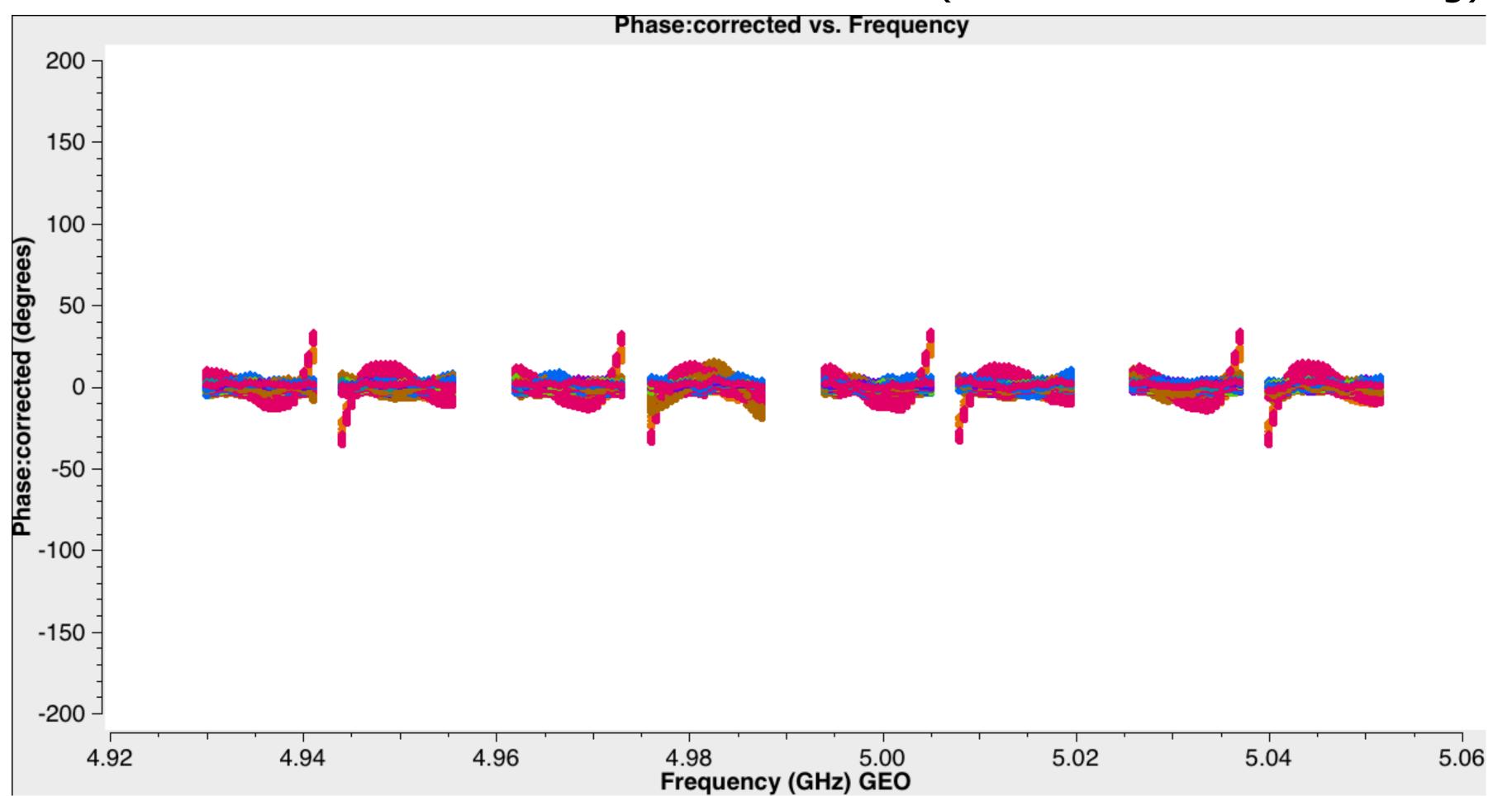
Multi-band delay solutions

- Noto telescope only shown here
- One solution per scan and spw combined
- Delay, phase, rate solutions primarily due to atmosphere



Multi-band fringe-fitting

Instrumental delays + multi-band delays (All baselines to Effelsberg)

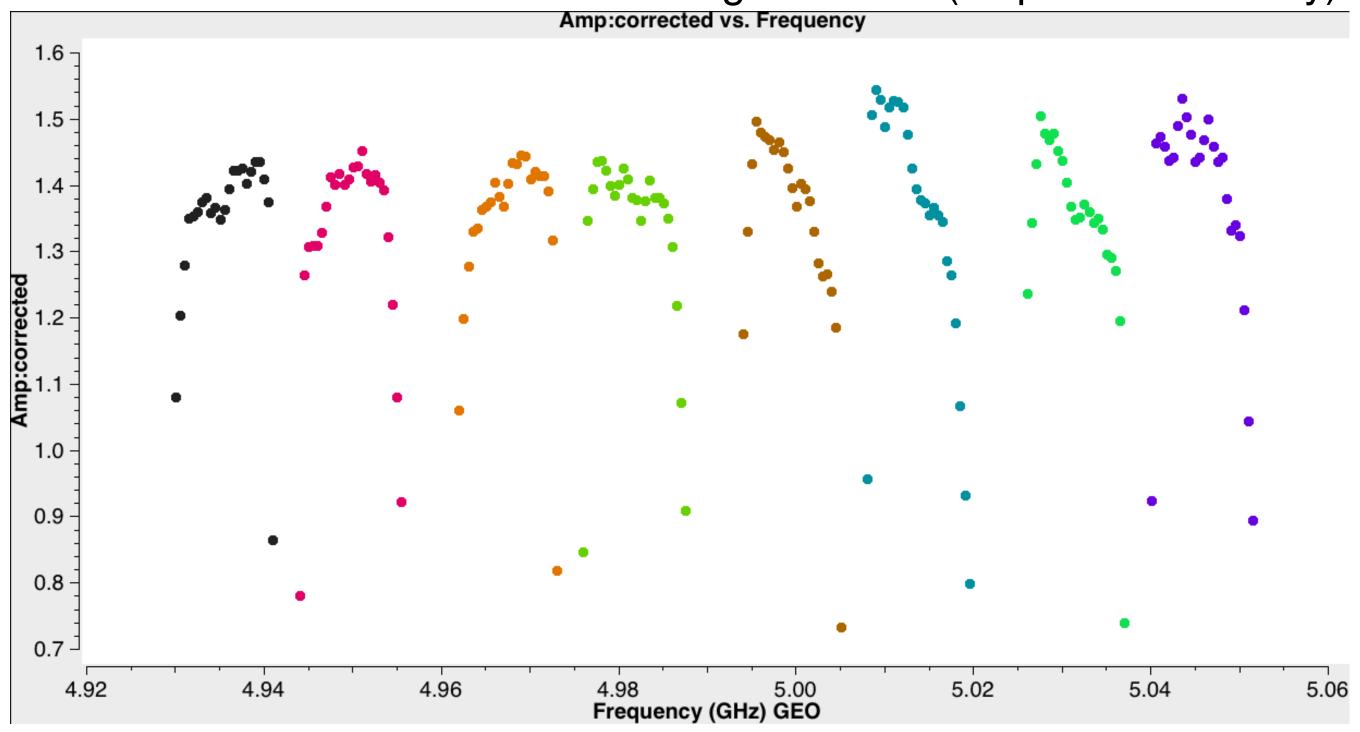


Bandpass correction

- Bandpass is the frequencydependent sensitivity across the observed frequency range.
- Variations are due to filters, receiver sensitivity variations & signal processing artefacts.
- Note you need to calibrate the phases & amplitudes first!
- Often done on the same scan that the sub-band delay is done on (corrected for phases & amps)

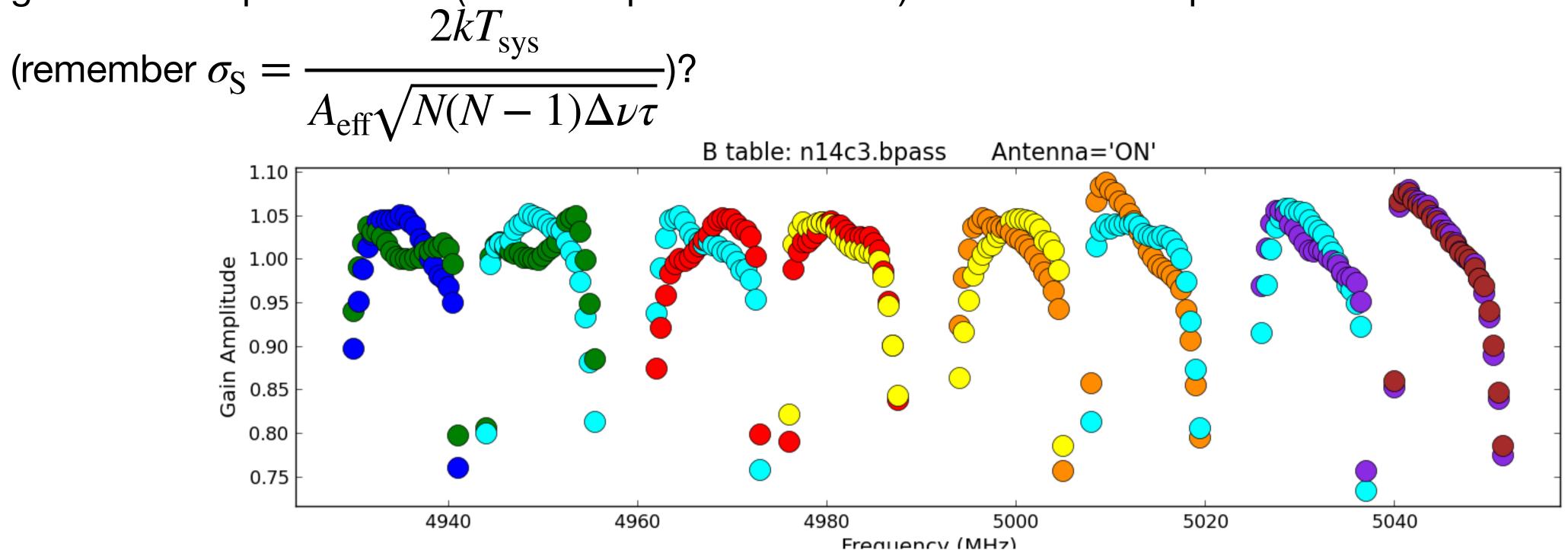
$$V_{pq}^{\text{obs}} = M_p B_p F_p G_p D_p E_p P_p T_p V_{pq}^{\text{true}} \dots$$

Baseline Effelsberg & Onsala (LL polarisation only)



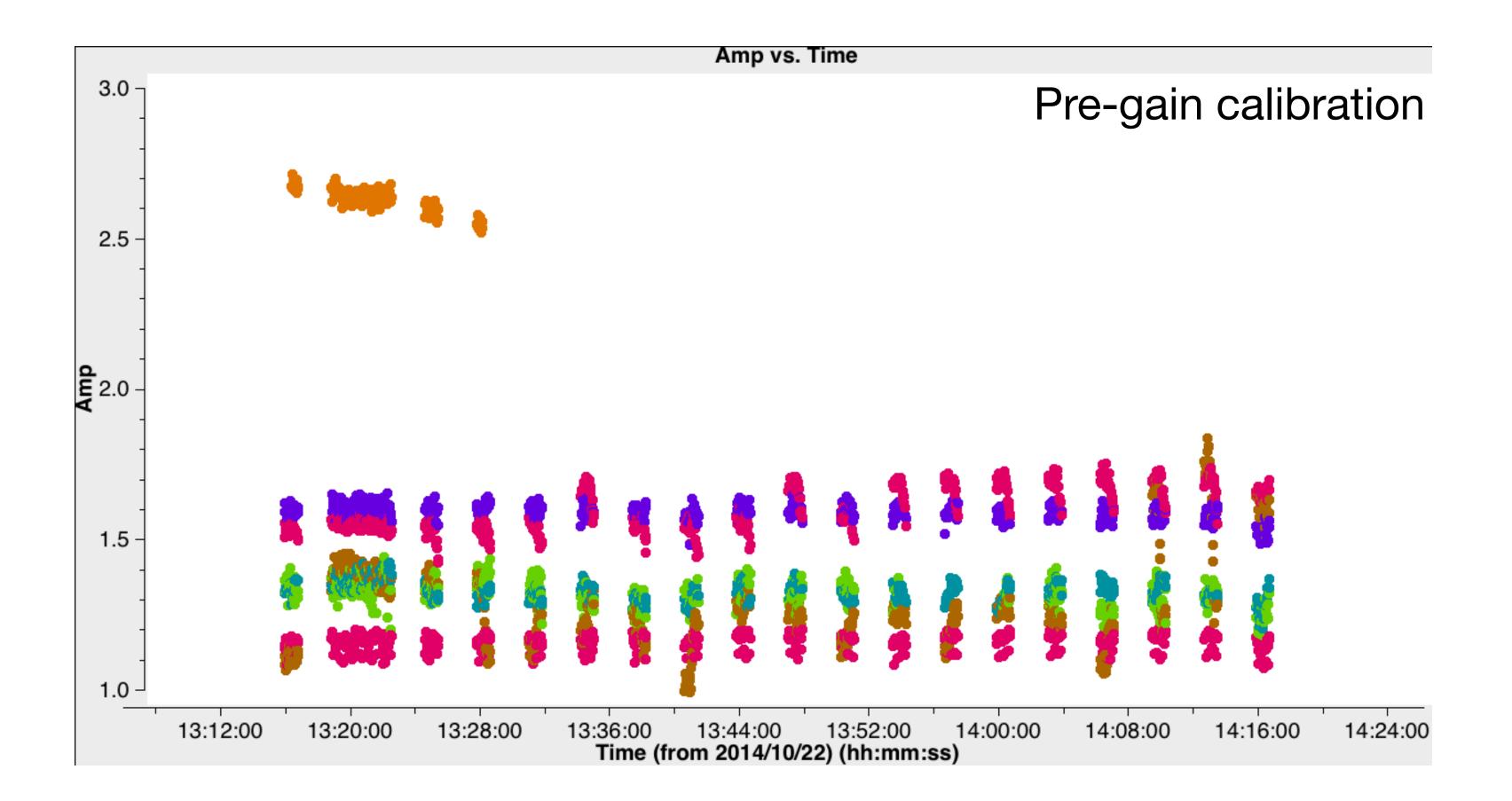
Bandpass correction

- Bandpass correction derives the amplitude and phases per antenna.
- Each antenna will have a distinctive amp vs frequency shape which can be derived from all baselines to that antenna! (It's a bunch of lots of simultaneous equations)
- Bandpass calibrators **must** be extremely bright (so can use the flux calibrator) as we need to get solutions per channel (and not spectral window!) to track the shape across the bandwidth



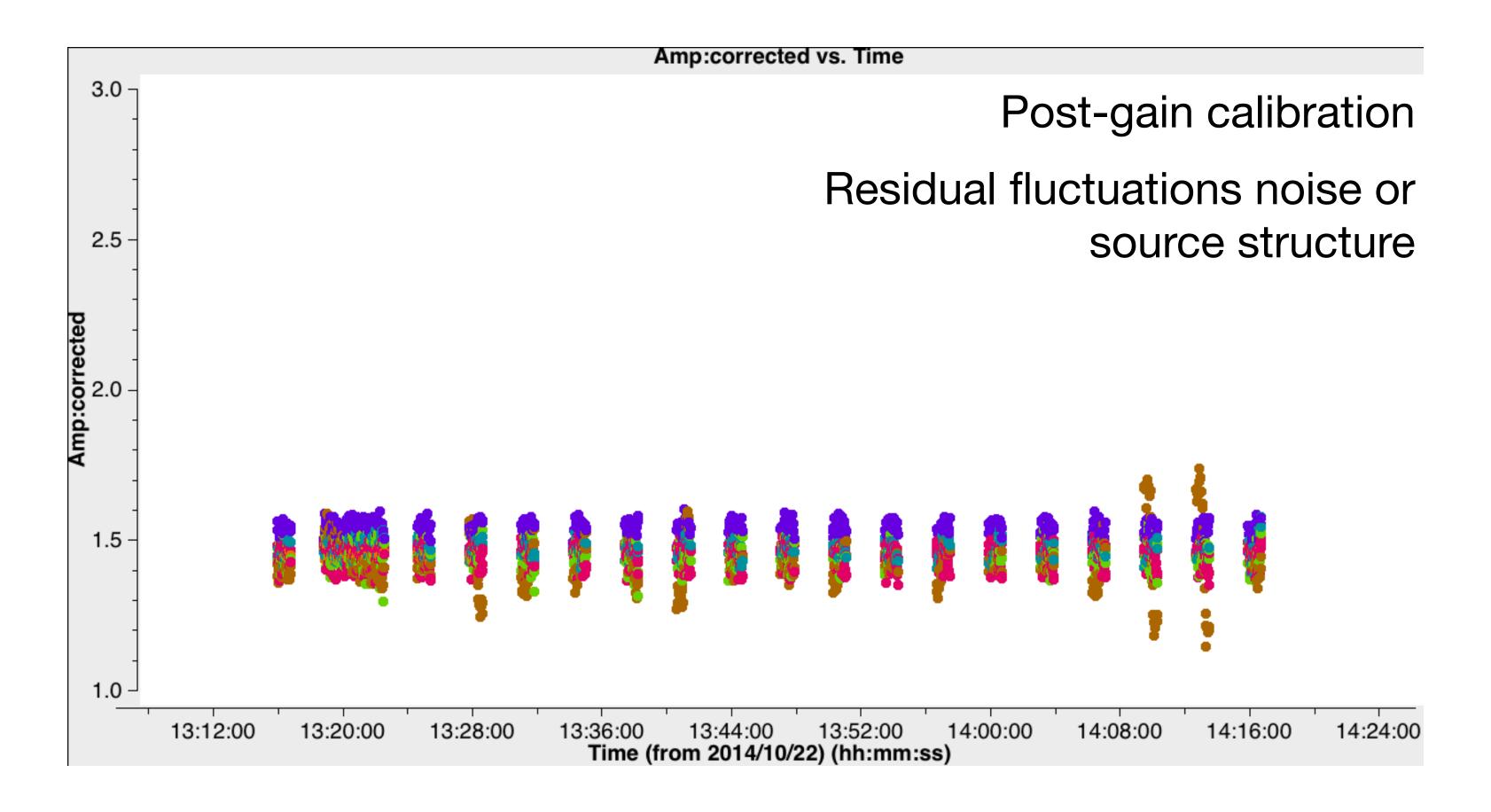
Phase referencing - amplitude

 The nodding between phase calibrator and target allows us to also track the amplitude variations, often caused by variable gains in the antenna system.



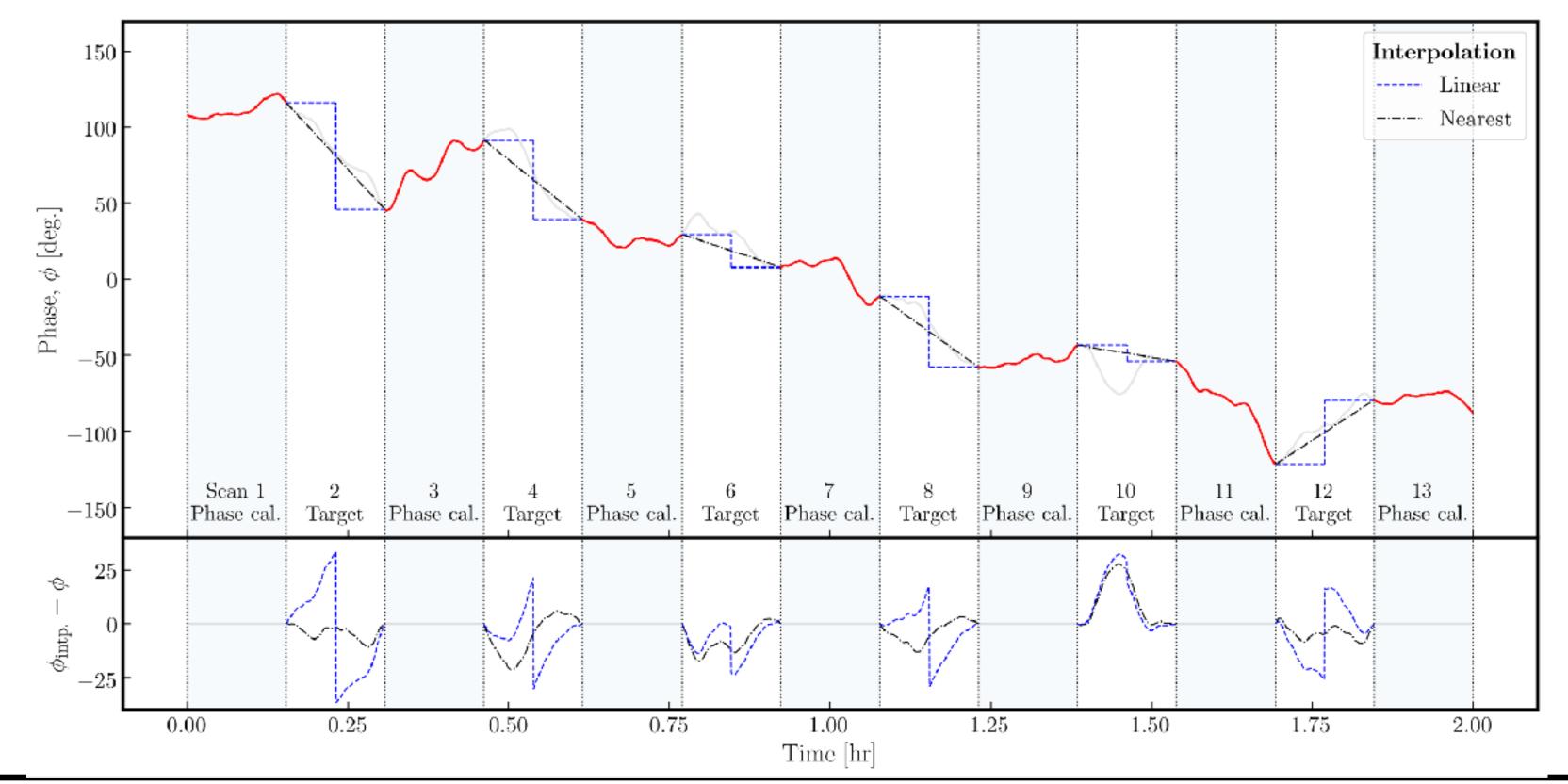
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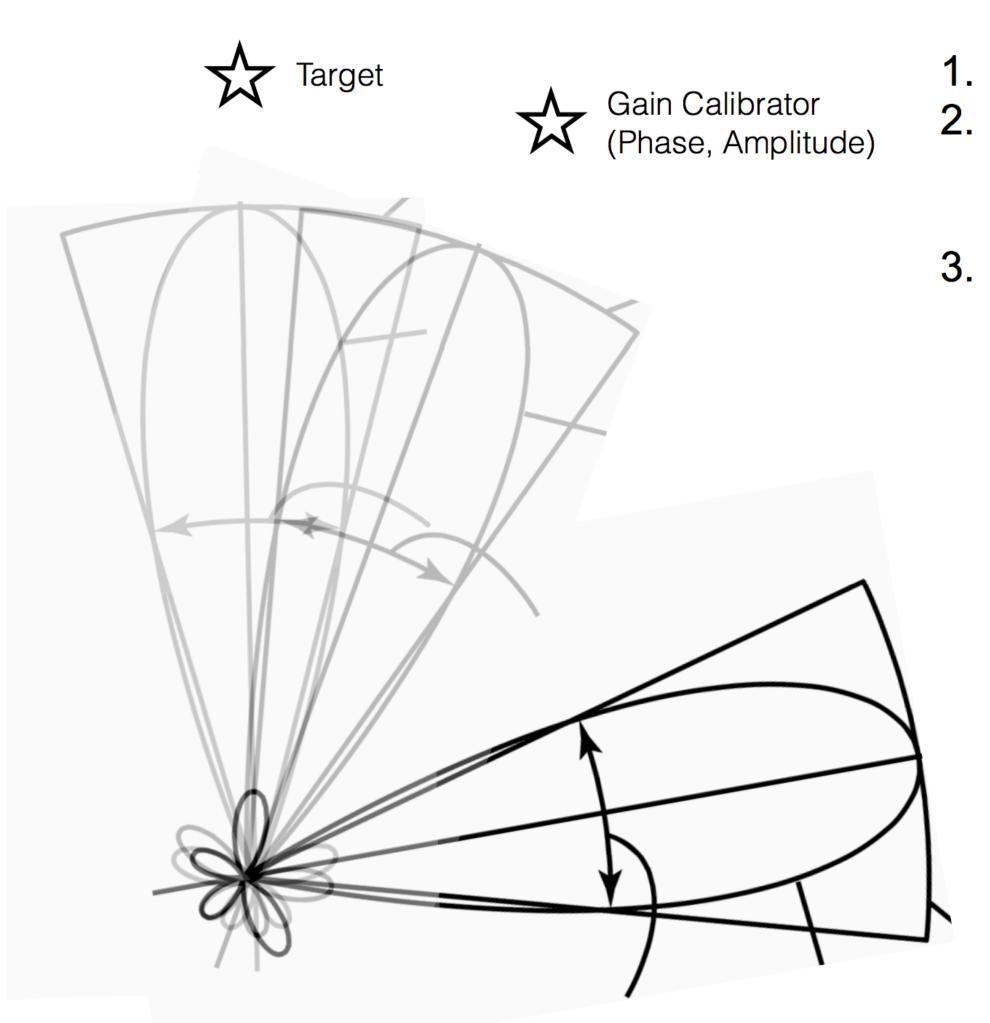


Transferring solutions

- As we are deriving various values with the various calibration techniques, we then need to interpolate these values in time and frequency to our target field!
- The solutions are not 100% correct just yet (e.g. the atmospheric paths are not the same) but we will explain in the self-calibration lectures.



Putting it all together - observing strategy



- 1. Observe **source**
- 2. Observe **calibrator** to measure gains (amplitude and phase) as a function of time.
- 3. Observe **bright calibrator** of known flux-density and spectrum to measure absolute flux calibration, band-pass and residual delays

Flux Calibrator (Flux, Bandpass, Delay)

Calibration strategy

*Remember to constantly look for bad data throughout!

- 1. Apply a priori calibration first (e.g., gaincurves, $T_{\rm sys}$, TEC, EOPs)
- 2. Sub-band fringe-fitting calibration on bright source remove instrumental delays.
- 3. Bandpass correction on same bright source
- 4. Multi-band fringe fitting on all phase reference (near to the target) sources removes time dependent phase errors (e.g., atmospheric errors)
- 5. Amplitude gain correction (using self-calibration)
- 6. Apply to target
- 7. Self-calibrate on target if bright enough.
- 8. Science!

And remember this works on the assumption that your phase calibrator is a point source at the phase centre! i.e. flat in amp and 0 in phase (unless you do self-calibration)