LOFAR Calibration

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LOFAR Data School

Funded by the European Union

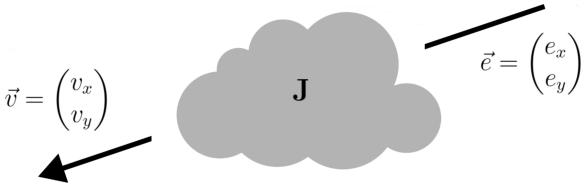
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Introduction

- Recap on radio interferometry fundamentals
- Radio Interferometry Measurement Equation
- Calibration fundamentals
- Calibration techniques

Radio interferometry

- Detector: Voltage signal $\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$
- Radio source: signal $\vec{e} = \begin{pmatrix} e_x \\ e_y \end{pmatrix}$
- Linked with Jones matrix: $\vec{v} = \mathbf{J}\vec{e}$



Jones Matrices

From sky signal to voltage: chain of Jones matrices
Each effect has its own Jones matrix

Note: Matrix multiplication in general not commutative!

$$\begin{aligned}\vec{v} &= \mathbf{J_1 J_2 J_3 ... J_n}\vec{e} \\ &= \mathbf{J}\vec{e}\end{aligned}$$

Foundational paper: Smirnov 2011

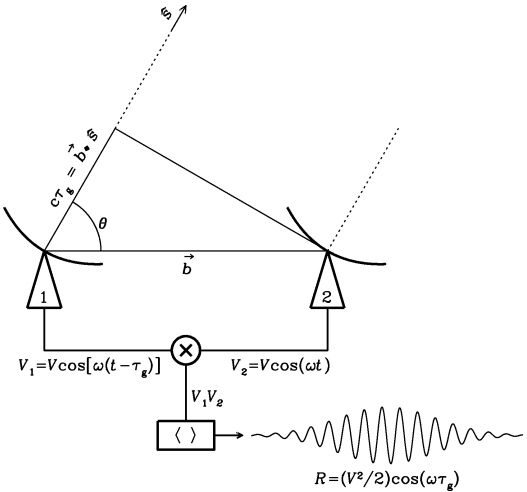
Jones formalism and visibilities (1)

Connection between the sky intensity distribution $I_\nu(l,m)$ and the *mutual coherence function* $\langle \vec{e}(\vec{r_1})\vec{e}(\vec{r_2})\rangle$: **Van Cittert-Zernike theorem**

$$\langle \vec{e}(\vec{r_1})\vec{e}(\vec{r_2}^{\,\dagger})\rangle \approx \iint I_\nu(l,m)e^{-2\pi i(ul+vm)}dldm$$

For full derivation: see e.g. Thompson, Moran, Swenson. Note that this means that the mutual coherence function and the sky intensity distribution are **Fourier pairs**!

Jones formalism and visibilities (2)



Condon and Ransom (1998)

Jones formalism and visibilities (3)

Mutual coherence function: assume quasi-monochromatic interferometer

- Two interferometers measure an electric field (real/cosine component):

$$V_1 = V\cos(\omega(t-\tau_g))$$
$$V_2 = V\cos(\omega t)\ ; \qquad \tau_g = \frac{\vec{b}\cdot\hat{s}}{c}$$

- Multiply, then take the time average (select for slow moving component)

$$\begin{aligned}\mathcal{R}_c = \langle V_1 V_2^\dagger \rangle &= \langle V^2\cos(\omega t)\cos(\omega(t-\tau_g))\\ &= \left\langle \frac{1}{2}V^2[\cos(2\omega t - \omega\tau_g) + \cos(\omega\tau_g)]\right\rangle\\ &= \frac{V^2}{2}\cos(\omega\tau_g)\end{aligned}$$

Jones formalism and visibilities (4)

Do the same for the sine-component, to construct the **complex visibility**:

$$\mathcal{V} = \mathcal{R}_c + i\mathcal{R}_s = Ae^{-i\phi}$$

If you have two feeds, such as an X- and Y-polarisation feed (as is the case with LOFAR), you get four visibilities per unique pair of antennas: $\mathcal{V}_{XX}, \mathcal{V}_{XY}, \mathcal{V}_{YX}, \mathcal{V}_{YY}$

This then forms the visibility matrix:

$$\mathbf{V}_{1,2} = \langle \vec{v_1} \vec{v_2}^{\dagger} \rangle = \begin{pmatrix} \mathcal{V}_{XX} & \mathcal{V}_{XY} \\ \mathcal{V}_{YX} & \mathcal{V}_{YY} \end{pmatrix} \propto \begin{pmatrix} I+Q & U-iV \\ U+iV & I-Q \end{pmatrix}$$

Radio Interferometry Measurement Equation

How does the Jones matrix play into this? Recall the Jones' equation : $\vec{v} = \mathbf{J}\vec{e}$

$$\mathbf{V}_{1,2} = \langle \vec{v}_1 \vec{v}_2^{\dagger} \rangle = \langle \mathbf{J}_1 \vec{e}_1 (\mathbf{J}_2 \vec{e}_2)^{\dagger} \rangle = \mathbf{J}_1 \langle \vec{e}_1 \vec{e}_2^{\dagger} \rangle \mathbf{J}_2^{\dagger} \equiv \mathbf{J}_1 \mathbf{B}_{1,2} \mathbf{J}_2$$

Radio interferometry Measurement equation (RIME):

$$\mathbf{V}_{i,j} = \mathbf{J}_i \mathbf{B}_{i,j} \mathbf{J}_j^{\dagger}$$

Calibration

- We create an image using $\mathbf{B}_{i,j}$, but we measure $\mathbf{V}_{i,j}$
- Calibration: determine Jones' matrices $\mathbf{J}_i$, so we can reconstruct $\mathbf{B}_{i,j}$
- Strategy: observe an object for which we already know how it looks like (Calibrator), so $\mathbf{B}_{i,j}$ is already known
- Minimize for Jones' matrix $\mathbf{J}$, for each station, time interval, frequency interval:

$$||\mathbf{V}_{\mathrm{obs},i,j} - \mathbf{J}_i \mathbf{V}_{\mathrm{model},i,j} \mathbf{J}_j^{\dagger}||$$

- For N stations, there will be N Jones' matrices, and $\frac{1}{2}N(N-1)$ visibilities (per time/frequency interval)

Calibration Strategies

- In practice, you need to minimize

$$||\mathbf{V}_{\mathrm{obs},i,j} - \mathbf{J}_i \mathbf{V}_{\mathrm{model},i,j} \mathbf{J}_j^{\dagger}|| + \epsilon_n$$

- Noise can become problematic, so we cannot solve the full Jones' matrix in one go
- Strategy: Restrict degrees of freedom (so only fit on-axis phases, amplitudes, etc.) or average over a larger time/frequency intervals
- Split effects in two categories:
  - *Systematic effects* caused by the instrument itself → Lecture by Cristina Cordun
  - *Ionospheric effects* that change rapidly in time/frequency/direction

Splitting the Jones Matrix

- Jones matrix is multiplication of a series of effects:
- Following elements:
  - $\mathbf{B}$: Bandpass
  - $\mathbf{G}$: Antenna gains (phase/amplitude)
  - $\mathbf{D}$: Polarisation alignment (cross-hand delays)
  - $\mathbf{E}$: Antenna beam pattern (Full-Jones!)
  - $\mathbf{P}$: Parallactic angle
  - $\mathbf{Z}$: **Ionospheric delay**
  - $\mathbf{F}$: **Faraday rotation**

$$\mathbf{J} = \mathbf{BGDEPZF}$$

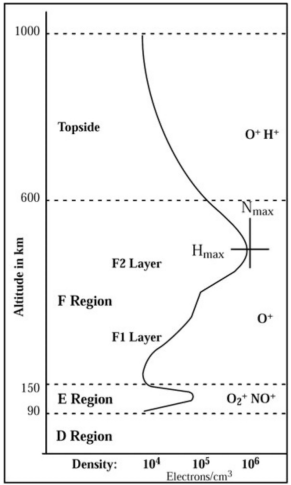
Antenna $\longleftarrow$ Ionosphere

Ionosphere

- Layer of the atmosphere, that contains free electrons
- Plasma, with an index of refraction that primarily depends on Total Electron Content (TEC)
- Index of refraction causes a (significant) delay:

$$\Phi_{ion} = -\frac{2\pi\nu}{c}\int_{\mathrm{LoS}}(n-1)\mathrm{d}l$$

- Refractive index is given by the Appleton-Hartree equation



Refractive index of the ionosphere

Delay is determined by refractive index:

$$\Phi_{ion} = -\frac{2\pi\nu}{c}\int_{\mathrm{LoS}}(n-1)\mathrm{d}l$$

Index of refraction is given by the Taylor expansion of the Appleton-Hartree equation:

$$n \approx 1 - \frac{q^2}{8\pi^2 m_e \epsilon_0}\frac{n_e}{\nu^2} \pm \frac{q^3}{16\pi^3 m_e^2 \epsilon_0}\frac{n_e B\cos\theta}{\nu^3} - \frac{q^4}{128\pi^4 m_e^2 \epsilon_0^2}\frac{n_e^2}{\nu^4} - \frac{q^4}{64\pi^4 m_e^3 \epsilon_0}\frac{n_e B^2(1-\cos^2\theta)}{\nu^4} + \mathcal{O}\left(\frac{1}{\nu^5}\right)$$

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Ionospheric delay, $\sim \frac{1}{\nu}$

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Second order, Faraday rotation, $\sim \frac{1}{\nu^2}$

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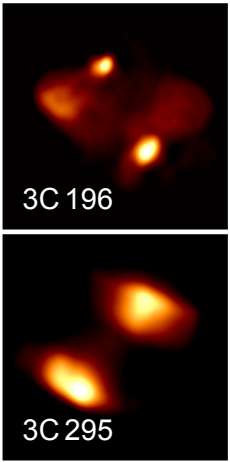
Index of refraction is given by the Taylor expansion of the Appleton-Hartree equation:

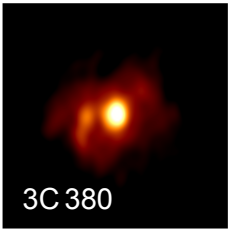
$$n \approx 1 - \frac{q^2}{8\pi^2 m_e \epsilon_0}\frac{n_e}{\nu^2} \pm \frac{q^3}{16\pi^3 m_e^2 \epsilon_0}\frac{n_e B\cos\theta}{\nu^3} - \frac{q^4}{128\pi^4 m_e^2 \epsilon_0^2}\frac{n_e^2}{\nu^4}$$
$$- \frac{q^4}{64\pi^4 m_e^3 \epsilon_0}\frac{n_e B^2(1-\cos^2\theta)}{\nu^4} + \mathcal{O}\left(\frac{1}{\nu^5}\right)$$

Higher order terms, only in extreme cases, $\sim \frac{1}{\nu^3}$

Calibration in practice

- Observe a calibrator source (3C 196, 3C 380, etc.)
- Calibrate to obtain systematic effects (see Systematic effects lecture by Cristina Cordun)
- Apply solutions to target data
- Solve for ionosphere on the target: Self-Calibration





Self-Calibration (1)

- Calibration when you do not know your model very well beforehand
- Iteratively improve both $\mathbf{J}_i$ and $\mathbf{V}_{\mathrm{model},i,j}$
- For N stations, there will be N Jones' matrices, and $\frac{1}{2}N(N-1)$ visibilities (per time/frequency interval)

$$||\mathbf{V}_{\mathrm{obs},i,j} - \mathbf{J}_i \mathbf{V}_{\mathrm{model},i,j} \mathbf{J}_j^{\dagger}|| + \epsilon_n$$

So, overdetermined system of equations!

Self-Calibration (2)

After applying systematic effects on the target data, perform self-calibration:

0. Get a reasonable starting model (from radio surveys, e.g. TGSS)
1. Convert model image to model visibilities $\mathbf{V}_{\mathrm{model},i,j}$ (predicting, e.g. with WSclean)
2. Find $\mathbf{J}_i$ such as to minimize $||\mathbf{V}_{\mathrm{obs},i,j} - \mathbf{J}_i \mathbf{V}_{\mathrm{model},i,j} \mathbf{J}_j^{\dagger}|| + \epsilon_n$
3. Correct your data with the Jones' matrices you just found: $\mathbf{V}_{\mathrm{cor},i,j} = \mathbf{J}_i^{-1} \mathbf{V}_{\mathrm{obs},i,j} \left(\mathbf{J}_j^{\dagger}\right)^{-1}$
4. Make an image of your corrected visibilities (e.g. with WSclean), which will also give you a model image
5. Go to (1)

Each iteration, you should improve your model, and $\mathbf{J}_i$ should become a better approximation of the ionospheric corrections required

Challenges (1)

- Self-calibration is critical for low-frequency interferometry
- Also, typically the hardest part:
  - Ionospheric effects move too fast in time/frequency (especially for low frequencies $< 80\mathrm{MHz}$)
  - No bright point source/calibrator in target field
  - Ionospheric effects change throughout the Field of View (especially for low frequencies)

Challenges (2)

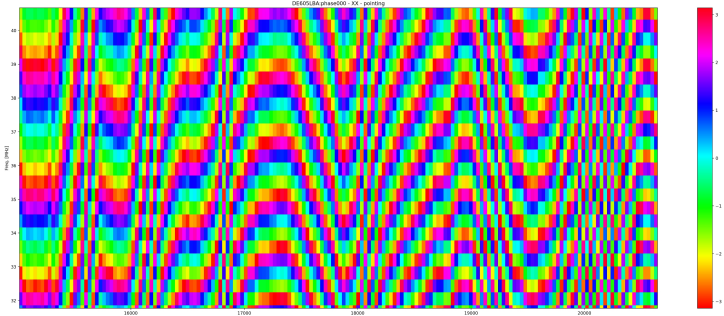
No bright point source/calibrator in target field

- Science target is rarely bright ($< 100\mathrm{Jy}$)
- Low flux density $\rightarrow$ Lower signal-to-noise
- Average more in time/frequency to increase signal-to-noise

Challenges (3)

Ionospheric effects move too fast in time/frequency

- Ionospheric effects move as $\Delta\phi_{\mathrm{iono}} \sim \frac{1}{\nu}$, at lower frequencies even as $\Delta\phi_{\mathrm{iono}} \sim \frac{1}{\nu^2}$
- If you average too much in time/frequency, your ionospheric phase changes too quickly within one ionspheric bin
- Rule of thumb: ~ 6 samples per full wrap



Challenges (4)

Ionospheric effects change throughout the Field of View

- Direction dependent calibration (after correcting for the bulk of the ionosphere)
- Split up field in *facets*, self-calibrate each facet individually
- Requires sufficient signal to noise per facet

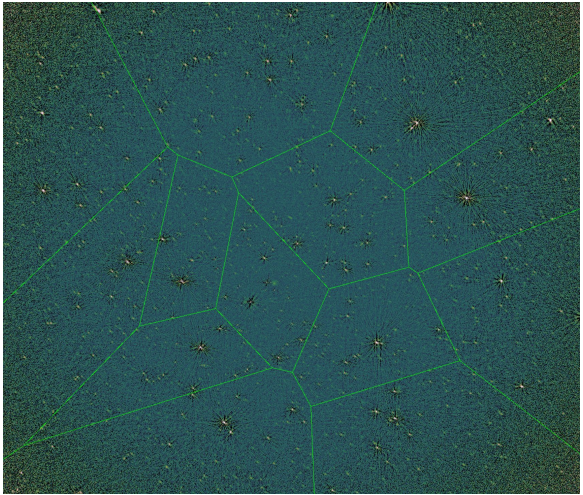
Facet self-calibration

Also known as 'direction-dependent calibration' or '3GC'

- Required if your science target is relatively faint, if there is a bright source in your field, or if you want a wide field image (e.g. galaxy clusters)
- Most survey results (LoTSS, LoLSS) offer direction dependent calibrated images
- Calibration pipelines that use a form of facet self-calibration:
  - DDF-pipeline (Tasse+21)
  - LiLF (de Gasperin+19,20,21), see LBA lecture (Jort Boxelaar)
  - Rapthor pipeline

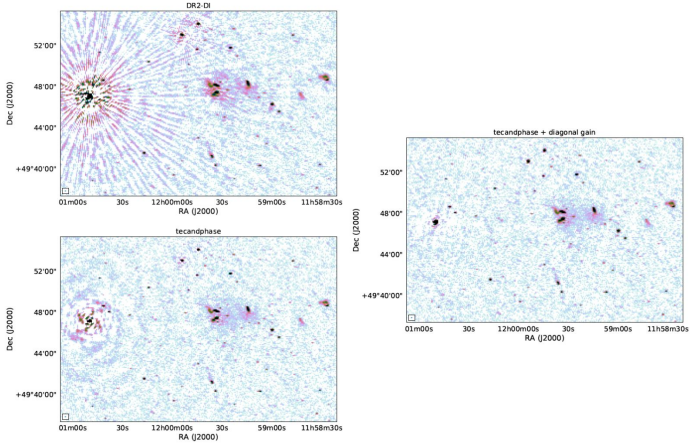
Facet self-calibration

- After applying direction independent solutions
- Solve for (primarily) ionospheric effects per direction/facet
- Basic idea:
  - Select facet
  - Subtract everything outside of the facet from data
  - Calibrate
  - Do this for all facets
- `facetselfcal` package (van Weeren+21)



Perturbative Calibration

- Phase effects: Asymmetric, stripy
- amplitude effects: Symmetric (typically less severe compared to phase)



Source: Reinout van Weeren

`https://github.com/rvweeren/lofar_facet_selfcal`

- Perturbative calibration tool: solve Jones' matrix in steps
- Wrapper over lower level calibration and imaging tools: `DP3` and `WSClean` and others
- Extract (single facet box on interesting science target)



Perturbative calibration scheme

Source: Van Weeren (+21)

- Start with biggest effects
- Continue with next biggest effect
- Solution interval / Smoothness-constraint (frequency)
- Arbitrary amount of perturbations without writing code

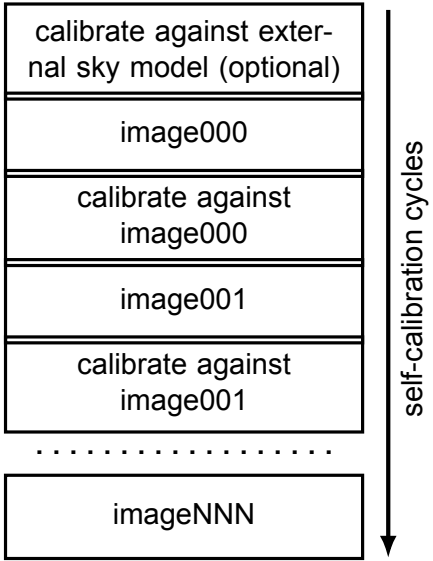
Very powerful for testing calibration strategies before implementing them in a pipeline!

Effects

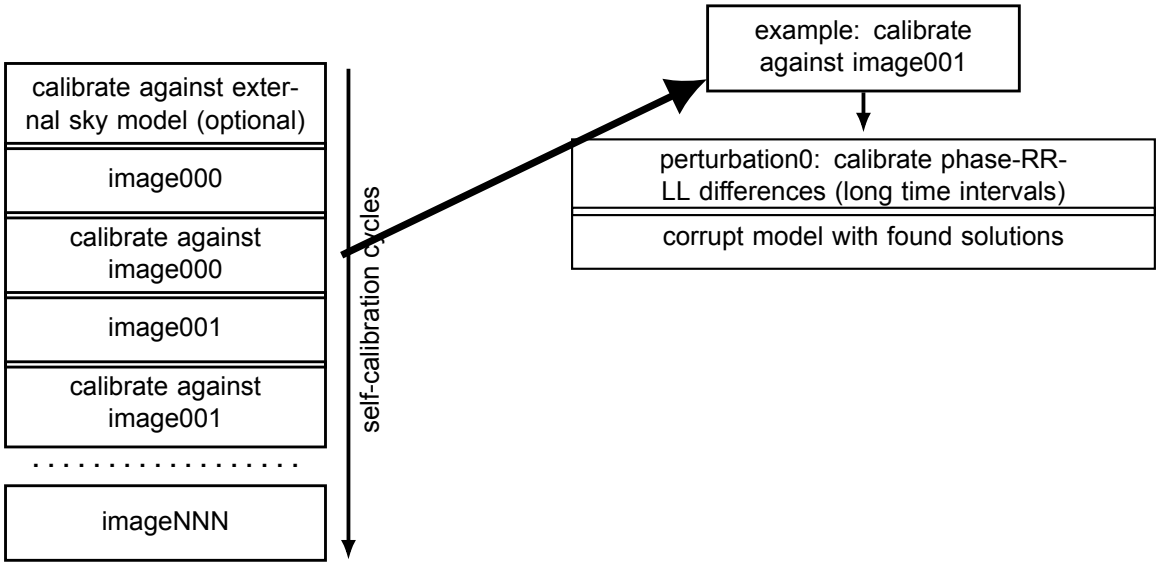
Most common types (Van Weeren +21):

- `scalarphase`: $e^{-i\phi}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$
- `scalaramplitude` $A\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$
- `scalarcomplexgain` $Ae^{-i\phi}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$
- `scalarphasediff` (in circular basis): $\begin{pmatrix}e^{-i\phi_{XX}} & 0\\ 0 & e^{-i\phi_{YY}}\end{pmatrix}$
- `phaseonly` $\begin{pmatrix}e^{-i\phi_{XX}} & 0\\ 0 & e^{-i\phi_{YY}}\end{pmatrix}$
- `amplitudeonly` $\begin{pmatrix}A_{XX} & 0\\ 0 & A_{YY}\end{pmatrix}$
- `fulljones` $\begin{pmatrix}A_{XX}e^{-i\phi_{XX}} & A_{XY}e^{-i\phi_{XY}}\\ A_{YX}e^{-i\phi_{YX}} & A_{YY}e^{-i\phi_{YY}}\end{pmatrix}$
- `tecandphase`: $e^{-i(\phi+\mathrm{TEC}/\nu)}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

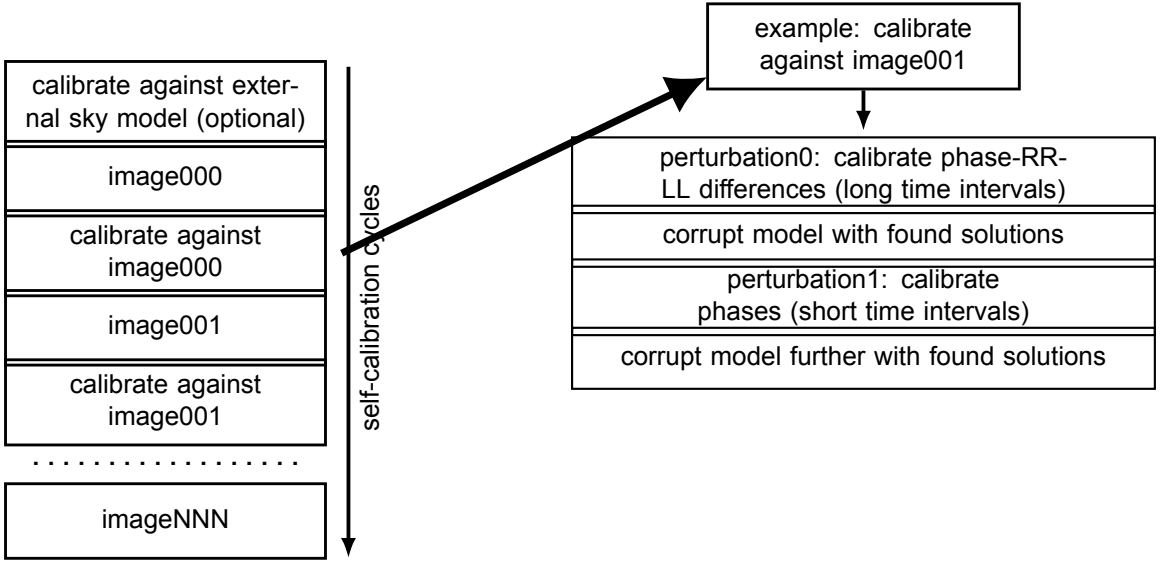
Perturbative solves in facetselfcal (van Weeren+21)



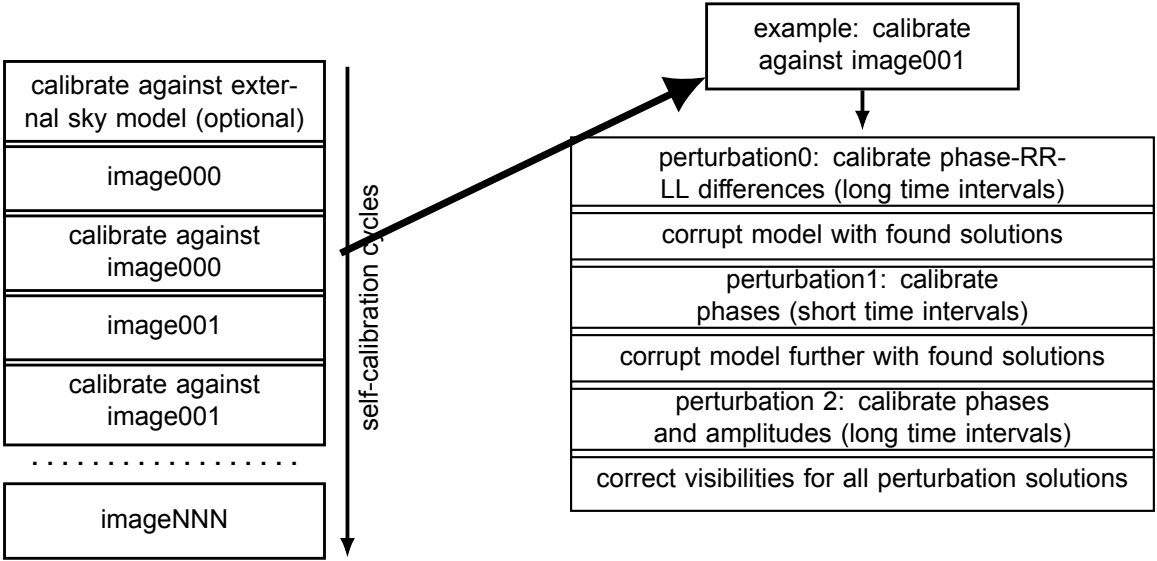
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Perturbative solves in facetselfcal (van Weeren+21)



Perturbative solves in facetselfcal (van Weeren+21)



Much more advanced options

- Automated selfcal (e.g. determining solints automatically)
- Automated facet calibration
- Polarisation
- Long baselines (Lecture by Frits Sweijen and Roland Timmerman)
- Other telescopes (MeerKAT, GMRT, ...)

Summary

- Calibration: finding Jones' matrices for $\mathbf{V}_{i,j} = \mathbf{J}_i \mathbf{B}_{i,j} \mathbf{J}_j^\dagger$
- Ionospheric effects:
  - Change rapidly in time/frequency/direction
  - Bulk component scales as $\phi \propto {}^{\mathrm{TEC}}/_{\nu}$
  - Higher order terms matter at low frequencies
- Facet self-calibration